Basic Course 10: Approximation of PDEs

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Overview. This course is organized as follows: 9 academic lessons, joint with 3 numerical labs with Matlab or FEniCS. Each slot lasts 2 hours.

The introduction (2 slots) deals with the principles of the Finite Difference Method (FDM) for Ordinary Differential Equations (ODEs) and 1D Partial Differential Equations (PDEs). The second part of the course (7 slots) consists in advanced material on the Finite Element Method (FEM) applied to elasticity, and on Mixed Finite Elements applied to incompressible fluid mechanics. In the last 5 slots, the last part of the course, numerical methods being used for the simulation of the propagation of electromagnetic waves are presented.

Part 1: Introduction

1-2. the Finite Difference Method (FDM) for Ordinary Differential Equations (ODEs), or Partial Differential Equations (PDEs) in 1D (heat, wave): stability, consistency, convergence.

Part 2: More on the Finite Element Method

3-4. The case of coupled system of PDEs: Example of the problem of linear elasticity, stemming from continuum mechanics: recalls and remainders on continuum mechanics, variational formulation, boundary conditions, existence and uniqueness of a solution. Example of finite element formulation for 2-D elasticity.

5. **Lab 1**: Finite element computations in 2-D elasticity (stress and strain)

6-7-8. Mixed Finite Element Method: an introduction on the Stokes equation (the Stokes equations govern the flow of a steady, viscous, incompressible, isothermal, Newtonian fluid). Variational formulation, link with constrained optimization, inf-sup condition (discrete and continuous). Example of inf-sup stable and inf-sup unstable finite element spaces.

9. **Lab 2**: Introduction to FEniCS, and Numerical solution of the Stokes equation with FEniCS.

Part 3: Discontinuous Galerkin Methods for Transient Wave Equations

10. Introduction to discontinuous Galerkin (DG) methods for the first-order linear hyperbolic systems: notion of admissible boundary conditions, weak formulation, energy identity.

11. Application to acoustic and electromagnetic problems: physical models, weak formulations, basis functions, construction of mass, stiffness and jump/flux matrices, time integration methods, L^2-stability, propagation in an unbounded domain.

12. **Lab 3**: Implementation of a Galerkin discontinuous method for an electromagnetic TE wave propagation in a 2D waveguide.

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