MINT Course "Inverted class"

P. Roesch and T.Fiedler - roesch@math.univ-toulouse.fr, fiedler@math.univ-toulouse.fr

This course is an "inverted class" in the sense that all lectures are given by students. The course is open to master students from M1 RI and M2 RI. This year the course will consist of two independent topics (6 weeks each), "Billards" and "Algebraic topology on smooth manifolds".

Abstract for the first part "Billards":

The mathematical problem of billiards is to investigate the behaviour of a trajectory of a ball on a billiard table. The ball moves and its trajectory is defined by the ball's initial position and its initial vector speed. The ball's reflections from the boundary of the table satisfy that the incidence and reflection angles are equal. Table with simple form like a circle, ellipse, triangle, rectangle, polygon...already exhibits a lot of interesting behaviours and lead to profound mathematical problems. The theory of billiards has many applications.

References:

- Utkir A. Rozikov " An introduction to Mathematical Billiards" World Scientific
- Serge Tabachnikov "Geometry and Billiards" Student Mathematical Library volume 20, American Mathematical Society (https://www.personal.psu.edu/sot2/books/billiardsgeometry.pdf)

Abstract for the second part "Algebraic topology on smooth manifolds":

We start by studying some basic notions of algebraic topology:

- Cell complexes and homotopy type
- Simplicial homology and exact sequences
- Cellular homology and mapping degrees.

We apply then these notions in order to study the topology of smooth compact manifolds:

- Critical points of smooth functions on manifolds and the Morse Lemma
- Morse functions and the homotopy type of smooth manifolds
- Morse inequalities and their applications
- Manifolds in Euclidien space and the Lefschetz theorem of hyperplan sections.

References:

The lectures will be based on chapters in three extremely well written books, which are for free on the internet:

1. A. Hatcher "Algebraic topology"
2. J. Milnor "Morse theory"
3. J. Milnor "Topology from the differentiable view point"