AN INTRODUCTION TO NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

FRANCIS FILBET

Abstract. In this course, we will first review some general stochastic systems of interacting particles with noise which are relevant as models for the collective behavior of animals. The first issue is to discuss how the mean-field limit the system is close to the solution of a kinetic partial differential equation. This study will include models widely studied in the literature such as the Cucker-Smale model, adding noise to the behavior of individuals. The difficulty, as compared to the classical case of globally Lipschitz potentials, is that in several models the interaction potential between particles is only locally Lipschitz, the local Lipschitz constant growing to infinity with the size of the region considered. In a second part, we will discuss the link with macroscopic models used in the literature and for numerical simulations of interacting particles.

Keywords. nonlinear partial differential equations, modelling

1. Introduction to kinetic theory

1.1. **Introduction.** Models describing self-organization of biological agents are currently receiving considerable attention. In this lectures, we will study a class of such models. More precisely, we focus on kinetic-type models for the flocking behavior exhibited by certain species of birds, fish, and insects.

1.2. Some examples of flocking models.

1.3. Kinetic and fluid description.

2. About kinetic equations

2.1. Introduction.

2.2. Existence of weak solutions.

2.3. Flocking behavior of kinetic models: quantitative estimates.

3. About fluid limit of kinetic equations

3.1. Introduction.

3.2. Relative entropy methods and Wasserstein distance.

4. Numerical methods

4.1. Particle methods.