ON BIOLOGICAL SCIENCE AND CLIMATE MODELLING

FRANCIS FILBET AND PASCAL NOBLE

Abstract. The first part of the course is dedicated to modelling in biologic science. In this part, we will first review some general stochastic systems of interacting particles with noise which are relevant as models for the collective behavior of animals. The first issue is to discuss how the mean-field limit the system is close to the solution of a kinetic partial differential equation. This study will include models widely studied in the literature such as the Cucker-Smale model, adding noise to the behavior of individuals. The difficulty, as compared to the classical case of globally Lipschitz potentials, is that in several models the interaction potential between particles is only locally Lipschitz, the local Lipschitz constant growing to infinity with the size of the region considered. In a second part, we will discuss the link with macroscopic models used in the literature and for numerical simulations of interacting particles.
The second part of the course is dedicated to modelling in climate science with a focus on ocean modelling. The aim of this course is to introduce some mathematical methods used in oceanography. We can identify two general classes of models with distinct applications: ocean climate models and water waves models. The first class describes ocean circulation at the scale of the planet and over climatologically relevant time scales. The second class of models describes the propagation of water waves at a regional scale on shorter time and spatial scales (from meter to a few kilometers) with application to forecasting and protection (submergence waves like tsnuamis, rogue waves, storm surges). We focus on the mathematical models leading to the derivation of hierarchies of models, the analysis of (linear and nonlinear) propagative properties of these models and typical problem associated to their numerical simulation.

Keywords. Collective dynamics, Oceanography, Modelling

1. Part I : biological science

1.1. Introduction to collective behavior.

Introduction.

Some examples of flocking models.

Kinetic and fluid description.

1.2. About kinetic flocking models.

Introduction.

Existence of weak solutions.

Flocking behavior of kinetic models: quantitative estimates.

1.3. About fluid limit of kinetic flocking models.

Introduction.

Relative entropy methods and Wasserstein distance.

1.4. Numerical methods.

2. Part II. Ocean Climate Models

2.1. Introduction.

Description of the relevant physical phenomenon and scales: density stratification, hydrostatic balance, Coriolis force.

Parametrization and non dimensionalization, Model derivation and simplifications.

Applications to the description of several ocean waves: gravity waves, barotropic (surface) Rossby waves , baroclinic (intern) Rossby waves, Gulf Stream.

2.2. **Water Waves Propagation.**

Description of the relevant physical scales and phenomenon: potential flows, dispersive effects, non-linear effects.

Parameterization and obtention of a hierarchy of models.

Applications to the description of several water waves: Russell solitary wave, Tsunamis, Tidale Waves (Mascaret).

References

1 Cucker, Felipe, and Steve Smale. "On the mathematics of emergence." Japanese Journal of Mathematics 2 (2007): 197-227.
2 Ha, Seung-Yeal, and Jian-Guo Liu. "A simple proof of the Cucker-Smale flocking dynamics and mean-field limit." (2009): 297-325.
3 Peszek, Jan. "Existence of piecewise weak solutions of a discrete Cucker-Smale's flocking model with a singular communication weight." Journal of Differential Equations 257.8 (2014): 2900-2925.
4 Lannes, David. The water waves problem: mathematical analysis and asymptotics. Vol. 188. American Mathematical Soc., 2013.
5 Dellar, Paul J., and Rick Salmon. "Shallow water equations with a complete Coriolis force and topography." Physics of fluids 17.10 (2005).