Algebraic Topology

Ricardo Campos

Course Summary

In topology, we are interested in understanding when two spaces can be continuously deformed into one another i.e. whether they are homotopy equivalent. Algebraic topology allows us to address such questions using algebraic tools. We associate to a topological space an algebraic object such as a group, vector space, algebra ..., which are algebraic "invariants" in the sense that they are isomorphic when spaces are homotopy equivalent.

This course provides an accessible introduction to the fundamental concepts and techniques in algebraic topology. We will begin by revisiting important concepts from topology and algebra and then discuss various algebraic invariants, such as homotopy groups and singular homology groups.

Topics Covered

1. Homotopy Equivalence
2. Fundamental Groups
3. Covering Spaces
4. Elements of Category Theory
5. Homological Algebra
6. Simplicial sets
7. Singular Homology
8. CW Complexes and Cellular Homology
9. Cohomology and Cup Products
10. (Co)homology with Coefficients
11. Higher Homotopy Groups

Prerequisites

Basic knowledge of point-set topology, group theory, and abstract algebra is strongly recommended. Attendance of the M1 course *Topologie et Algèbre* will be helpful but not required. We will review essential materials as needed.

References

- *Algebraic Topology* by Allen Hatcher. Available for free at `http://www.math.cornell.edu/~hatcher/AT/ATpage.html`
- An elementary illustrated introduction to simplicial sets `https://arxiv.org/pdf/0809.4221.pdf`