

M2 Introductory Course Proposal

First Semester 2024-2025

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Title: “Introduction to Complex Analytic Geometry”

Abstract: The first part of the course will give the basic notions and results in complex analysis of several variables:

- holomorphic functions of several variables, the Cauchy formula and its consequences (especially the Cauchy inequalities);
- harmonic and (pluri)subharmonic functions;
- currents and their basic properties.

The second part of the course will present the notions of complex manifold and complex structure, positivity properties of certain differential forms and currents on complex manifolds, Hermitian metrics on these manifolds (especially the basic properties of Kähler metrics), together with the basic cohomologies (De Rham, Dolbeault, Bott-Chern, Aeppli) and some rudiments of Hodge theory (especially the Hodge isomorphism in the general compact Hermitian case and the Hodge decomposition/symmetry in the compact Kähler case).

The third part of the course will cover the notions of vector bundles, connections and curvature on complex manifolds and will give their main properties. If time permits, some positivity notions for vector bundles and classical vanishing theorems will also be presented.

References.

[Dem 97] J.-P. Demailly — *Complex Analytic and Algebraic Geometry*—<http://www-fourier.ujf-grenoble.fr/~demailly/books.html>

[GH94] P. Griffiths, J. Harris — *Principles of Algebraic Geometry* — Wiley Classics Library, 1994.

[Pop20] D. Popovici — *Non-Kähler Hodge Theory* — book available at <https://www.math.univ-toulouse.fr/~popovici/>