Introduction to Spectral Theory

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In this course we will discuss basic tools of spectral theory. This generalizes in infinite dimension the analysis of the spectrum and the diagonalization of matrices, and the applications to linear ODEs. Spectral theory is used in all the areas of mathematics where linear opearators arise (PDEs, probability, numerical analysis, geometry, etc.).

1. Linear operators - Spectrum
- Spectrum of bounded operators
- Unbounded operators, closed operators
- Spectrum of unbounded operators - Resolvent
- Discrete spectrum, Riesz projections
- Operators and quadratic forms - Representation theorem

2. Adjoint - Selfadjoint operators
- Adjoint of an operator
- Bounded normal operators
- Symmetric operators, selfadjoint operators
- Spectral properties of selfadjoint operators
- Essential spectrum
- Min-max principle

3. Compact operators - compact resolvents
- Compact operators
- Spectrum of compact operators
- Operators with compact resolvents
- Relatively compact perturbation - Weyl's Theorem

4. Semigroups and evolution equations
- Strongly continuous semigroups
- Dissipative operators
- Generator of a semigroup
- Hille-Yosida Theorem