Basic course A5 : Sobolev spaces & Elliptic equations

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This course is about second order elliptic equations, whose prototype is the Poisson equation

$$\frac{\partial^2 u}{\partial x_1^2} + \cdots + \frac{\partial^2 u}{\partial x_n^2} = f \quad \text{in } \Omega \subset \mathbb{R}^n \,,$$

for a function u of n variables in an open set Ω, also subject to appropriate boundary conditions at $\partial\Omega$, and a given function f. This and similar equations have many applications in physics and in other areas of mathematics. The goal of the course is to establish existence, uniqueness and regularity properties of solutions.

1 Classical solutions

Smooth solutions: representation formulas, maximum principles, Harnack inequalities.

2 Sobolev spaces

Fundamental tools about the spaces $W^{1,p}(\Omega)$ of functions with weak derivatives in $L^p(\Omega)$: Sobolev and Poincaré inequalities, compactness properties, boundary traces.

3 Weak solutions

Existence, uniqueness and regularity results for weak solutions of general elliptic equations in divergence form. Applications to problems in the Calculus of Variations.

References

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[3] Han, Q., and Lin, F. *Elliptic partial differential equations*, 2nd ed. ed., vol. 1 of *Courant Lect. Notes Math.* New York, NY: Courant Institute of Mathematical Sciences; Providence, RI: American Mathematical Society (AMS), 2011.

[4] Krylov, N. V. *Lectures on elliptic and parabolic equations in Hölder spaces*, vol. 12 of *Grad. Stud. Math.* Providence, RI: AMS, American Mathematical Society, 1996.

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