Introduction to Partial Differential Equations

Mihai MARIŞ

Many phenomena in physics, chemistry or biology are modelled by partial differential equations (PDE). This course provides a review of the most common PDE and their applications. Its main goal is to introduce the mathematical tools widely used in the theory of PDE.

1. Crash course: Functional analysis, distributions and Fourier transforms.

Weak topologies, reflexive spaces, L^p spaces.

Lax-Milgram Theorem.

Compact operators.

Distributions.

Schwartz' space. Tempered distributions. Fourier transform of tempered distributions.

2 Sobolev spaces.

$W^{1,p}(\Omega)$ and $H^k(\Omega)$, where Ω is an open domain.

Sobolev spaces via Fourier transform: $H^s(\mathbf{R}^N)$.

Sobolev embedding theorems. Poincaré inequalities.

Weak formulation of elliptic boundary value problems.

3. Elliptic equations.

Laplace's and Poisson's equations. Modelling, representation formulae and qualitative properties. Maximum principle. Harnack inequalities.

Elliptic operators in divergence form. Weak solutions. Regularity of weak solutions.

Some basic examples of nonlinear elliptic problems.

4. Introduction to evolution equations.

Linear parabolic and dispersive equations.

Resolution of the linear heat equation, Schrödinger equation and wave equation in $\mathbf{R}^N$ by using the Fourier transform.

Semigroups of linear operators. Hille-Yosida theorem.

Duhamel's formula. Nonlinear equations.

References

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