Stochastic calculus and Markov processes

The Brownian Motion is a random phenomenon which plays a fundamental role in the theory of stochastic processes. Due to a strongly irregular dynamics, the construction of integrals with respect to this process needs the development of a specific (stochastic) integration theory. In the first part of the course, we will then focus on this topic generally called stochastic calculus (or Ito calculus) going from the probabilistic construction of integrals with respect to continuous martingales towards the study of Stochastic Differential Equations (SDEs), processes which are now widely used in modelling. Under reasonable assumptions, solutions of SDEs are Markov processes. In the second part, we will thus focus on the general theory of Markov Processes including its basics and its applications to the study of SDEs. We will in particular show that the Markov processes are at the intersection of several topics by establishing the link between Martingales and Partial Differential Equations. If possible, we will end the course by some results about the long time behavior of such dynamics.

Outline:

1. Brownian Motion and Stochastic Calculus
   - Brownian Motion
   - Martingales and Semimartingales
   - Stochastic Integrals
   - It Formula and Applications
   - Stochastic Differential Equations
2. Markov Processes
   - Definition, Semi-group, Construction, Strong Markov property.
   - Feller property, Existence of a cdlg version.
   - Infinitesimal Generator, Martingale Problems, Feynman-Kacs formula.
   - Stationarity, Existence and Uniqueness of Invariant Distributions.

References

1. K.L. Chung, R.J. Williams, Introduction to stochastic integration.
2. Ethier S., Kurtz T., Markov processes, characterization and convergence.
3. I. Karatzas, S. Shreve, Brownian motion and stochastic calculus.
4. J.F. Le Gall, Mouvement brownien, martingales et calcul stochastique.
5. D. Revuz, M. Yor, Continuous martingales and Brownian motion.
6. L.C.G. Rogers, D. Williams, Diffusions, Markov processes and martingales.
7. J.M. Steele, Stochastic calculus and financial applications.