Thesis proposal

On-line change detection on Lie groups for satellite attitude control

1 Context

Detecting abrupt changes consists in identifying the instants when the statistical behavior of a system changes significantly. This problem is crucial in a wide range of engineering applications, where faults, environmental changes, or operational mode switches may occur. In a real time setting, detecting such changes must be performed on-line, i.e. whenever a sensor provides information on the system. Different approaches have been proposed for this purpose over the past 50 years. However, in many problems, systems can be described by variables submitted to some geometrical constraints that classical approaches fail to take into account. For instance, in the context of autonomous navigation, a dynamic vehicle is equipped with accelerometers and gyrometers that, after some pre-processing, yield the position and the attitude of the vehicle, i.e. its orientation with respect to a reference frame. It takes the form of either a rotation matrix or a quaternion, both of which take values on specific spaces called Lie groups [8]. A variety of algorithms has recently been introduced to infer such parameters on-line while preserving their properties. Most of them are variants of the Kalman filter [1][2]. Nevertheless, in the case of highly maneuvering vehicles or due to external constraints, they may experience value jumps that should be detected, not only to ensure seamless navigation, but also to obtain valuable information on the mobile and its environment.

2 Targeted application

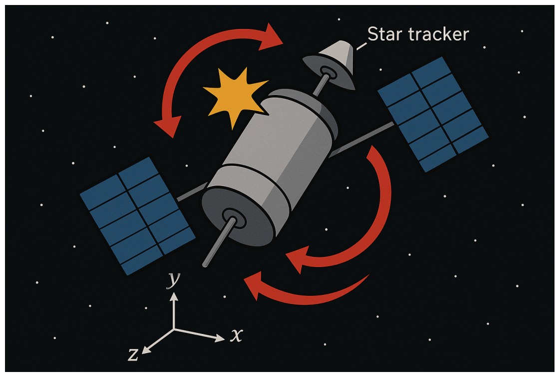

Figure 1: Illustration of the attitude control problem

In this thesis, we focus on satellite attitude control which consists in maintaining the orientation of a satellite in space [4]. This problem is of paramount importance, for instance in communication systems to ensure reliable communication with Earth: antenna misalignment can indeed seriously impair data transmission. Also, in remote sensing, Earth imaging sensors embedded on satellites must be oriented toward an observation area. Generally, this orientation is measured by a star sensor [3], often hybridized with an Inertial Measurement Unit (IMU). In situations where a satellite experiences some failures, its attitude can suddenly change and it is therefore essential to detect these variations.

3 Objectives of the thesis

This thesis aims to develop new methods for on-line change detection when the data and/or the parameters of interest lie on matrix Lie groups. The latter have the specificity of being equipped with both a Riemannian and a group structure [7]. They make it possible to leverage mathematical tools (especially from the fields of statistics and optimization). Practical examples are the set of rotation matrices and the set of orthonormal matrices.

In this thesis, we focus on systems whose behavior can be captured through assumed dynamic models. Their unknown states are then sequentially inferred from the observations by a filter, usually a Kalman or particle filter [6]. Two classes of approaches will be investigated for change detection:

- multiple-model based methods,
- statistical-test based methods.

In the first case, several candidate evolution models are considered and the main challenge lies in identifying the instants when a transition occurs from one dynamic model to another. Among state-of-the art methods, we can cite Jump State Markov Model (JSMM) or Interacting Multiple Model (IMM) which are formulated in a Bayesian filtering framework [9, 5, 7]. Although these methods are well established in the literature, they are dedicated to Euclidean variables and the objective of this thesis will be to generalize them within a Lie group framework. In the second setting, the outputs of the filtering algorithm are monitored to flag any deviations in their statistical behavior. Usually, residuals defined as the difference between the actual observations and their predictions are leveraged. Classical detection tests designed for that purpose are dedicated to probability distributions defined on vector-spaces. Again, the objective will be to adapt them to Lie-group defined quantities.

The algorithms developed will be theorized and then numerically validated on our satellite attitude control problem: in the case of maneuvers, a dynamic change on the orientation or/and the position of the satellite can be indeed modeled on the Lie groups $SO(3)$ or $SE(3)$. By simulating synthetic observations on $SO(3)$ provided by a star sensor, the proposed framework will be tested. If possible, the approaches will also be applied to real star sensor data.

4 General informations

- Skills: statistical signal processing, recursive estimation, Kalman filtering theory. Knowledge about information geometry will be an added-value.
- Programming tools: MATLAB and /or PYTHON.
- Required curriculum: MSc. or Eng. diploma in applied mathematics, computer sciences or statistics.
- **Starting period: October 2026.**
- Applications (CV, motivation letter, grades, recommendation letter) and informal inquiries are to be e-mailed to Samy Labsir (Associate Professor, IPSA/TéSA), `samy.labsir@ipsa.fr` and Audrey Giremus , `audrey.giremus@u-bordeaux.fr`, (Full Professor, University of Bordeaux).
- Location: TéSA laboratory, Toulouse, France and IMS Laboratory, Talence, France.
- **Salary: 1500 euros net per month.**

References

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