Introduction to Symplectic Topology

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Symplectic topology is the study of the *global* phenomenon occuring in symplectic geometry. Symplectic geometry arose as the geometry of classical mechanics, but nowadays sits like a somewhat mysterious spider in the centre of a spectacular web of links, interactions, and cross fertilisations with many other fields, among them algebraic, complex, contact, convex, enumerative, Kähler, Riemannian and spectral geometry, dynamical systems (Hamiltonian dynamics, ergodic theory, mathematical billiards), non-linear functional analysis, PDEs, number theory and combinatorics. Symplectic embeddings of simple shapes like (collections of) balls, ellipsoids, and cubes lie at the heart of symplectic topology ever since Gromov's seminal Nonsqueezing theorem from 1985. Symplectic embedding results give a feeling for what 'symplectic' means, and together with the techniques used in their proofs lead to new connections to other fields, including those mentioned above.

Prerequisite

The course of Algebraic and Differential Topology. Notions of Differential Geometry.

Programme

We shall mostly follow [3] and [1] and cover at least the following: Linear symplectic geometry, Symplectic manifolds, Almost complex structures, Symplectomorphisms, and Symplectic invariants (in particular Floer homology)

References

[1] M. Audin and M. Damian, Théorie de Morse et homologie de Floer, Savoirs Actuels (Les Ulis), EDP Sciences, Les Ulis; CNRS Éditions, Paris, 2010

[2] J. Lee, Introduction to smooth manifolds, Graduate Texts in Mathematics 218, Springer, 2003.

[3] D. McDuff and D. Salamon, Introduction to Symplectic Topology, Oxford Mathematical Monographs, The Clarendon Press Oxford University Press, 1998.