Introduction to Homotopy Theory

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Course Summary

Classical homotopy theory is the study of topological spaces up to homotopy equivalence. Roughly, two spaces are homotopy equivalent if one can be "continuously deformed" into the other. For example, a cylinder can be flattened to a circle. This type of situation does not only arise in topology: in mathematics, one often tries to understand when two "objects" (algebraic, geometrical, topological, ...) are equivalent. Modern approaches to homotopy theory provide a more axiomatic description of such situations, allowing one to study objects "up to equivalence" in a more systematic way.

In this course, we introduce the theory of model categories to study the classical homotopy theory of topological spaces and its generalizations. We provide examples of model categories (chain complexes, simplicial sets, . . .) and tools to compare them. Finally, we apply this formalism to obtain a purely algebraic description of the rational homotopy theory of topological spaces.

Topics Covered

1. Homotopy theory of topological spaces
2. Model categories
3. Quillen equivalences
4. Chain complexes
5. Simplicial sets and topological spaces
6. Cofibrantly generated model structures and transfer
7. Commutative differential graded algebras
8. Sullivan minimal models
9. Rational homotopy theory and commutative differential graded algebras
10. Applications and computations.

Prerequisites

Attendance of the basic M2 course *Algebraic Topology* is strongly recommended. We will review essential materials as needed.

References

- *Homotopy theories and model categories* by William G. Dwyer and Jan Spaliński. Available for free at `https://math.jhu.edu/~eriehl/616/DwyerSpalinski.pdf`
- *Model categories* by Mark Hovey. Available for free at `https://people.math.rochester.edu/faculty/doug/otherpapers/hovey-model-cats.pdf`
- *Rational homotopy theory* by Yves Félix, Stephen Halperin et Jean-Claude Thomas. Available at `https://link.springer.com/book/10.1007/978-1-4613-0105-9`
- *Rational homotopy theory. A brief introduction* by Kathryn Hess. Available for free at `https://arxiv.org/pdf/math/0604626.pdf`