Syllabus – Advanced course
Reaction-Diffusion Equations

- **Main goal:** In this course, we introduce some basic tools and discuss some recent progress on the study of reaction-diffusion models motivated by spatial ecology, evolution and epidemiology. We will focus on one class of equation, namely the scalar Fisher-KPP equation for which we will present a self-content analysis of the long time dynamics of its solutions. Such an equation arises not only in the present context, but also in the study of the Brownian motion.

- **Contents:** We will provide an overview of some techniques used in the study of partial differential equations of reaction-diffusion type, and we will discuss the derivation of the models. More specifically, there will be five different parts in these lectures.

  Part 1: We will start by an introduction of the equations and the models under study, and explain the motivations coming from spatial ecology, evolution and epidemiology. We will also we explain why reaction-diffusion equations are a natural and relevant way to model biological invasions, that is, processes in which a living species, of the animal, vegetal or bacterial type, settles in a given environment.

  Part 2: We will review (and/or introduce) some basic techniques for the study of elliptic and parabolic equations such as maximum principles, notions of sub/super-solutions, regularity theory, etc... We will also deal with the existence, uniqueness and regularity of the solutions of the Cauchy problem.

  Part 3: We will study some special entire solutions of the equation which take the form of traveling fronts. We will discuss their existence, uniqueness, qualitative properties together with some of their stability properties.

  Part 4: We will study the long time behavior of the solutions of the Cauchy problem starting from compactly supported initial data, and characterize so-called spreading properties of the solutions.

  Part 5: Depending on time, we will show how our theoretical results can be applied in some specific contexts (e.g. spatial spreading of an epidemic) and discuss extensions of our models. A non exhaustive list of possible extensions is: road/field models, non local diffusion or reaction mechanisms, models with multiple species (e.g. spatially extended Lotka-Volterra systems), ecological models in periodic or hostile environments, etc...

- **Prerequisite:** We (strongly) encourage students to take the course A4 on *Elliptic PDEs and evolution problems.*

- **Perspectives:** This advanced course could serve as the basis for a PhD on related subjects.

- **Key-words:** parabolic PDEs, traveling waves, invasion phenomena, spreading properties.

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