

M2R Course: Regularization of Ill-posed Inverse Problems and Applications

PIERRE MARÉCHAL

Introduction and Course Objectives

Inverse problems arise in numerous applications, from signal and image processing to statistics and partial differential equations. However, many such problems are ill-posed, necessitating regularization techniques to ensure stable and meaningful solutions. This course aims to:

- Provide a comprehensive understanding of ill-posed problems in a Hilbertian framework.
- Introduce variational regularization methods, including Tikhonov and generalized Tikhonov regularization.
- Highlight the variational version of mollification.
- Explore applications in signal-image processing, statistics, and partial differential equations.

Course Content

1. Introduction to Inverse Problems

- Definition and examples of inverse problems.
- Well-posedness and ill-posedness (Hadamard criteria).

2. Regularization Techniques

- Tikhonov regularization: classical and generalized forms.
- Variational regularization approaches.
- Approximate inverses.
- Variational mollification techniques.
- A glance at nonlinear methods.

3. Applications

- Signal and image processing.
- Statistical estimation problems.
- Inverse problems in partial differential equations.

Prerequisites

- Basic knowledge of functional analysis.
- Fourier analysis.

References

- [1] David L. Colton and Rainer Kress. *Inverse acoustic and electromagnetic scattering theory*. 4nd. Springer, 2013.
- [2] HW Engl. “Regularization of Inverse Problems”. In: *Mathematics and its Applications* 375 (1996).
- [3] Andreas Kirsch et al. *An introduction to the mathematical theory of inverse problems*. Vol. 120. Springer, 2011.
- [4] V.A. Morozov. *Methods for Solving Incorrectly Posed Problems*. Springer, 1984.