

Random matrices and free probability theory

Guillaume Cébron and Mireille Capitaine

Random matrix theory is a very active field of mathematics and physics. It has emerged from the applications, first in data analysis (Wishart 1928) and later in nuclear physics (with the celebrated Wigner's semicircle theorem 1955). During the last decades, it turned out to be linked with various branches of mathematics. In particular Voiculescu discovered in 1991 a deep connection between the so-called free probability theory that he developed in the 80's and random matrix theory. This lecture aims to give a flavor of this relationship.

We will first present an introduction to random matrices and provide a proof of Wigner's semicircle law. Then we will introduce free probability theory and show that the semicircle law also appears as the limit in a free central limit theorem. Finally, we will explain how free probability theory plays a fundamental role in the analysis of the spectrum of random matrix models in large dimension.

References

- [1] Hiai, Fumio and Petz, Dénes. *The semicircle law, free random variables and entropy*. Mathematical Surveys and Monographs, 77. American Mathematical Society, Providence, RI, 2000.
- [2] Mingo, James A. and Speicher, Roland. *Free probability and random matrices*. Fields Institute Monographs, 35. Springer, New York; Fields Institute for Research in Mathematical Sciences, Toronto, ON, 2017.
- [3] Voiculescu, Dan. *Lectures on free probability theory*. Lectures on probability theory and statistics (Saint-Flour, 1998), 279–349, Lecture Notes in Math., 1738, Springer, Berlin, 2000.