M2RI – Reading Seminar 2

Part I

Scaling limits in statistical mechanics

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Part II

Unique continuation for second order elliptic partial differential equations

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Part I: Scaling limits in statistical mechanics

Introduction

Statistical mechanics studies the properties of large particle ensembles where particles interact with each other according to the laws of mechanics. Since the systems are typically very large (for example, in a gas there are around 10^{23} molecules in a unit of volume), the derivation of reduced models describing the statistics of the system, such as the so-called **kinetic models**, has became a central problem in the field.

In this seminar, we will study some mathematical methods to obtain kinetic models through **many-particle limits**, such as the so-called Boltzmann-Grad limit. We will focus on models in which particles interact locally, by binary interactions. In particular, we will outline the main steps of the derivation of some classical kinetic equations, namely

- the Smoluchowski coagulation equation for polymerization and aerosol growth
- and the Boltzmann equation for rarefied gases.

Possible topics and references

We will study between 2 to 4 topics chosen among the ones proposed below.

- **The Boltzmann-Grad limit**

  1. From the deterministic hard-sphere system to the Boltzmann equation ([1] and [2], Section 4.6)
  2. From coagulating Brownian particles to the Smoluchowski coagulation-diffusion equation ([4], Section 4)

- **Pairwise interactions in spatially homogeneous models**

  3. From the Kac's model to the Boltzmann equation ([2], Section 4.1)
  4. From coalescing particles to the Smoluchowski coagulation equation [3]

Prerequisites

Functional analysis or stochastic processes.

References

[1] C. Saffirio. Derivation of the Boltzmann equation: hard spheres, short-range potentials and beyond. In: From Particle Systems to Partial Differential Equations III: Particle Systems and PDEs III, Braga, Portugal, (2016) 301–321. Springer International Publishing.

[2] L. P. Chaintron, and A. Diez. Propagation of chaos: A review of models, methods and applications II. *Kinetic and Related Models* 15.6 (2022): 1017-1173.

[3] M. Escobedo, F. Pezzotti. Propagation of chaos in a coagulation model. Mathematical Models and Methods in Applied Sciences, 23(06) (2013) 1143–1176.

[4] R. Lang and N. X. Xanh. Smoluchowski's theory of coagulation in colloids holds rigorously in the Boltzmann-Grad-limit. *Z. Wahrscheinlichkeitstheor. verw. Geb.*, **54** (3) (1980) 227-280.

Part II: Unique continuation for second order elliptic partial differential equations

This reading seminar is dedicated to the study of *Unique Continuation Properties* (UCP) for second order elliptic partial differential equations. A typical and well-known example of such UCP is the following: let Ω be a non-empty connected open domain of $\mathbb{R}^d$, and ω be a non-empty open subset of Ω. Then any harmonic function u that is identically zero in ω is identically zero in Ω. Schematically:

$$\begin{cases} \Delta u = 0 \text{ in } \Omega, \\ u = 0 \text{ in } \omega, \end{cases} \quad \Rightarrow u = 0 \text{ in } \Omega.$$

We will demonstrate that result, discuss some extensions, in particular the so-called *propagation of smallness* which is a quantified version of the UCP, and some applications of UCP to inverse problems and spectral theory.

References

[1] Jérémi Dardé and Sylvain Ervedoza. A short introduction to Carleman estimates. Control of partial differential equations, 1â79. Ser. Contemp. Appl. Math. CAM, 24, 2023. HAL version.

[2] Michael V. Klibanov and Alexandre A. Timonov. *Carleman estimates for coefficient inverse problems and numerical applications.* Inverse and Ill-Posed Problems Series. (2004).

[3] Jérôme Le Rousseau, Gilles Lebeau and Luc Robbiano. *Elliptic Carleman estimates and applications to stabilization and controllability. Volume I. Dirichlet boundary conditions on Euclidean space.* Progress in Nonlinear Differential Equations and Their Applications 97, Subseries in Control, BirkhÃ¤user, 2022.

[4] Sergio Vessella. Notes on unique continuation properties for Partial Differential Equations – Introduction to the stability estimates for inverse problems. Preprint arXiv, 2023.