Part I

Scaling limits in statistical mechanics

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Part II

Unique continuation for elliptic partial differential equations

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Part I

Introduction

Statistical mechanics studies the properties of large particle ensembles where particles interact with each other according to the laws of mechanics. Since the systems are typically very large (for example, in a gas there are around 10^{23} molecules in a unit of volume), the derivation of reduced models describing the statistics of the system, such as the so-called **kinetic models**, has became a central problem in the field.

In this seminar we will study some mathematical methods to obtain kinetic models through **many-particle limits**, such as the mean-field and the Boltzmann-Grad limits. In particular, we will outline the main steps of the derivation of some classical kinetic equations, namely

- the Vlasov equation (1938), used in the modelling of plasmas,
- the Smoluchowski equation (1916) for polymerization and aerosol dynamics
- the Boltzmann equation (1872) for rarefied gases
- or the Hartree equation (1927) used in quamtum chemistry.

Possible topics and references

We will study between 2 to 4 topics chosen among the six topics listed below.

- **Mean-field limits**

  In the case of systems in which the behaviour of each particle is affected by all the other particles, the mean-field limit allows to replace the full system by one effective equation for one typical particle under the effect of an average interaction.

  Possible topics on the derivation of kinetic equations using mean-field limits are:

  1. From Newton to Vlasov equation: mean-field limit in classical mechanics ([3], Lecture 1)
  2. Derivation of the Smoluchowski coagulation equation from coalescing particles ([5], Section 4)
  3. From Schrödinger to Hartree equation: mean-field limit in quantum mechanics ([3], Lecture 3)

- **Pairwise interactions and the Boltzmann-Grad limit**

  If instead the particles only interact locally, for example, in the case of pairwise collisions in rarefied gas, then the mean-field theory is not suitable. Alternative approaches are available involving the so-called Boltzmann-Grad limit.

  Possible topics and references in this direction are:

  4. From the deterministic hard-sphere system to the Boltzmann equation ([2], Section 1.5 and [1], Section 4.6)
  5. From the Kac's model to the Boltzmann equation ([1], Section 4.1)
  6. From coagulating Brownian particles to the Smoluchowski coagulation-diffusion equation ([4], Section 4)

Prerequisites

Functional analysis or stochastic processes.

References

[1] L. P. Chaintron, and A. Diez. Propagation of chaos: A review of models, methods and applications II. *Kinetic and Related Models* 15.6 (2022): 1017-1173.

[2] F. Golse. On the dynamics of large particle systems in the mean field limit. *Lecture Notes in Applied Mathematics and Mechanics*, vol 3. Springer, Cham (2016) 1-144.

[3] F. Golse, Mean-field limits in statistical dynamics. *arXiv preprint* (2022) arXiv:2201.02005.

[4] R. Lang and N. X. Xanh. Smoluchowski's theory of coagulation in colloids holds rigorously in the Boltzmann-Grad-limit. *Z. Wahrscheinlichkeitstheor. verw. Geb.*, **54** (3) (1980) 227-280.

[5] J. R. Norris, Smoluchowski's coagulation equation: Uniqueness, nonuniqueness and a hydrodynamic limit for the stochastic coalescent. *Ann. App. Probab.* (1999) 78-109.

Part II

This reading seminar is dedicated to the study of *Unique Continuation Properties* (UCP) for second order elliptic partial differential equations. A typical and well-known example of such UCP is the following: let Ω be a non-empty connected open domain of $\mathbb{R}^d$, and ω be a non-empty open subset of Ω. Then any harmonic function u that is identically zero in ω is identically zero in Ω. Schematically:

$$\begin{cases} \Delta u = 0 \text{ in } \Omega, \\ u = 0 \text{ in } \omega, \end{cases} \quad \Rightarrow u = 0 \text{ in } \Omega.$$

We will demonstrate that result, discuss some extensions, in particular the so-called *propagation of smallness* which is a quantified version of the UCP, and some applications of UCP to inverse problems and spectral theory.

References

[1] Jérémi Dardé and Sylvain Ervedoza. A short introduction to Carleman estimates. Control of partial differential equations, 1â79. Ser. Contemp. Appl. Math. CAM, 24, 2023. HAL version.

[2] Michael V. Klibanov and Alexandre A. Timonov. *Carleman estimates for coefficient inverse problems and numerical applications.* Inverse and Ill-Posed Problems Series. (2004).

[3] Jérôme Le Rousseau, Gilles Lebeau and Luc Robbiano. *Elliptic Carleman estimates and applications to stabilization and controllability. Volume I. Dirichlet boundary conditions on Euclidean space.* Progress in Nonlinear Differential Equations and Their Applications 97, Subseries in Control, BirkhÃ¤user, 2022.

[4] Sergio Vessella. Notes on unique continuation properties for Partial Differential Equations – Introduction to the stability estimates for inverse problems. Preprint arXiv, 2023.