Reading Seminar: Entropy and Large Deviations

Introduction

The central theme of this reading seminar is the notion of *Entropy* and its connection with the *Theory of Large Deviations*. Specifically, we shall study the *Boltzmann–Gibbs–Shannon (BGS) entropy* associated with a finite probability distribution $(p_1, \ldots, p_n)$, as well as its various generalizations, such as the *Rényi entropy*, the *relative entropy*, and the *relative Rényi entropy*. Both the BGS entropy and the relative entropy possess intuitive and elegant axiomatic characterizations, which will be discussed in detail.

We shall motivate Rényi entropies via the *Large Deviation Principle (LDP)* in probability theory. The connection between LDP and notions of entropy is profound and plays a central role in this seminar. For this reason, *Cramér's theorem* will be proven, along with the more involved proof of *Sanov's theorem*.

Another important aspect is the *Fisher entropy* and its role in parameter estimation. Historically, these topics have been studied primarily in statistics, but they may also play a significant role in the study of entropy in *non-equilibrium statistical mechanics*.

Overall, this seminar aims to explore the multiple facets of entropy at the intersection of information theory, statistics, and statistical mechanics, highlighting fundamental principles and the deep links between probability and entropy.

Main References

1. Vojkan Jakšić, *Lectures on Entropy. Part I*, arXiv:1806.07249.

2. Dembo, A., & Zeitouni, O. (1998), *Large Deviations: Techniques and Applications*, 2nd edition, Springer.