Proposal for a Master 2 advanced course
Hyperbolic initial boundary value problems and numerical schemes

The study of hyperbolic systems is motivated by numerous physical applications : electromagnetism, elastodynamics, fluid mechanics and so on. For all these problems, numerical simulation is a valuable tool in industrial contexts because of the lack of a thorough understanding of the solutions to these partial differential equations. Numerical simulations usually require an appropriate match between the numerical scheme that is used inside the computational domain and the numerical boundary conditions that are implemented.

The aim of this advanced course is to study the theory of numerical boundary conditions for some approximations of systems of partial differential equations of the form :

$$\frac{\partial u}{\partial t} + \sum_{j=1}^{d} A_j \, \frac{\partial u}{\partial x_j} = 0 \, .$$

The sessions will first deal with the discretization of such evolution problems by means of finite difference schemes in the whole space (without any physical nor artificial boundary). The influence of numerical boundary conditions and the various notions of stability one can adopt will then be studied when the domain is a half-space. A major goal of the course is to give a complete characterization of what is known as *strong stability* for numerical schemes with boundary conditions. The characterization relies on an algebraic type condition that is first due to Gustafsson, Kreiss and Sundström in the fully discrete case.

Prerequisite : the students need a good knowledge of Fourier analysis and some basic knowledge of matrix theory. It is suitable (but not mandatory) to have some basic knowledge of transport/wave equations.

Some references : S. Benzoni-Gavage et D. Serre, *Multi-dimensional hyperbolic partial differential equations : first-order systems and applications*, Oxford University Press, 2007.

J.-F. Coulombel, Stability of finite difference schemes for hyperbolic initial boundary value problems, *HCDTE Lecture Notes. Part I. Nonlinear Hyperbolic PDEs, Dispersive and Transport Equations*, American Institute of Mathematical Sciences, 2013.

B. Gustafsson, H.-O. Kreiss et J. Oliger, *Time dependent problems and difference methods*, John Wiley & Sons, 1995.

B. Gustafsson, H.-O. Kreiss et A. Sundström, Stability theory of difference approximations for mixed initial boundary value problems II, *Mathematics of Computation*, 1972.

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