M2 Dissertation Topic 2024-2025

Title: Invariance of the plurigenera of projective manifolds of general type under holomorphic projective deformations

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Description: The main goal is to study in depth the proof given by Y.-T. Siu in [Siu98] to the following long-conjectured result:

Let $\pi : \mathcal{X} \longrightarrow \Delta$ *be a holomorphic* **projective** *family of compact complex manifolds* $X_t := \pi^{-1}(t)$ *of general type over the open unit disc* $\Delta \subset \mathbb{C}$. *Then, for every positive integer* m, *the* m*-th plurigenus* $p_m := dim_{\mathbb{C}} H^0(X_t, mK_{X_t})$ *of* X_t *is independent of* $t \in \Delta$.

One denotes by K_Y the canonical bundle of a complex manifold Y. By the family $\pi : \mathcal{X} \longrightarrow \Delta$ being *projective* one means that there exists a positive holomorphic line bundle on its total space $\mathcal{X}$. A compact complex manifold X is said to be of general type if its canonical bundle K_X is *big* (a positivity assumption).

Siu went on to prove in [Siu02] the above result without the *general-type* assumption on the fibres X_t of the family, but work on this dissertation will focus on the general-type case which differs in one key respect from the general case.

A key ingredient in the proof is the Ohsawa-Takegoshi L^2 extension theorem of [OT87]: given a complete Kähler manifold (X, ω), a complex submanifold $Y \subset X$ and a holomorphic section f on Y of an appropriately positive holomorphic line bundle $L \longrightarrow X$ such that f satisfies a certain L^2 condition, there exists a holomorphic extension F of f to X whose L^2-norm is bounded above by a uniform constant multiple of the L^2-norm on Y of the original f. The philosophy and the techniques of the proof are well explained in [Siu96].

Besides the Ohsawa-Takegoshi L^2 extension theorem, the proof of [Siu98] uses the theory of multiplier ideal sheaves of Nadel, whose relevant details are well explained in [Dem00].

Prerequisites: basic notions in several complex variables (holomorphic and (pluri)subharmonic functions, currents, elliptic differential operators) and in complex geometry (complex manifold, complex structure, Hermitian metrics, connections on and curvature of holomorphic vector bundles, basic notions of Hodge Theory).

References.

[Dem97] J.-P. Demailly — *Complex Analytic and Algebraic Geometry*—http://www-fourier.ujf-grenoble.fr/ demailly/books.html

[Dem00] J.-P. Demailly — *Multiplier Ideal Sheaves and Analytic Methods in Algebraic Geometry* — Lectures given at the ICTP School "Vanishing Theorems and Effective Results in Algebraic Geometry" held in Trieste, Italy, April 24 – May 12, 2000.

[OT87] T. Ohsawa, K. Takegoshi — *On The Extension of* L^2 *Holomorphic Functions* — Math. Z. **195** (1987) 197-204.

[Siu96] Y.-T. Siu — *The Fujita Conjecture and the Extension Theorem of Ohsawa-Takegoshi* — in *Geometric Complex Analysis* ed. Junjiro Noguchi *et al*, World Scientific Publishing Co. 1996, pp.

577 - 592.

[Siu98] Y.-T. Siu — *Invariance of Plurigenera* — Invent. Math. **134** (1998), 661-673.

[Siu02] Y.-T. Siu — *Extension of Twisted Pluricanonical Sections with Plurisubharmonic Weight and Invariance of Semipositively Twisted Plurigenera for Manifolds Not Necessarily of General Type* — in *Complex Geometry* (Göttingen, 2000), 223-277, Springer, Berlin, 2002.

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Further details: I will be available from March to July 2025 to supervise an M2 student working on this topic. Moreover, work on this dissertation can be a springboard for future PhD work that I will be glad to supervise.