M2 Dissertation Topics 2025-2026

The prerequisites and the references are the same for the two topics proposed below, the second one being a continuation of the first. A student may initially choose the first topic and, if time permits, segue into the second.

Topic 1

Title: Multiplier ideal sheaves and the Ohsawa-Takegoshi L^2 Extension Theorem

Supervisor: Dan Popovici

Description: Nadel's multiplier ideal sheaves are a key tool in the study of singularities of plurisubharmonic (psh) functions and Hermitian fibre metrics on holomorphic line bundles. The two main results that will be studied are the coherence of these sheaves and Nadel's Vanishing Theorem, a far-reaching generalisation of Kodaira's Vanishing Theorem.

The other key result in the L^2 theory in complex analysis and geometry is the extension theorem given in its first form by Ohsawa and Takegoshi in [OT87] and then generalised in a series of papers by Ohsawa, as well as by Manivel, Siu, Demailly and others who also gave a host of geometric applications, one of which is Siu's proof of the invariance of the plurigenera in projective holomorphic families of compact complex manifolds. This last result is covered by the second topic proposed below.

The Ohsawa-Takegoshi L^2 Extension Theorem is the third fundamental result in the theory of L^2 estimates in complex analytic geometry, the earlier two ones being Hörmander's L^2 estimates (for solutions u of the equation $\bar\partial u = v$) and Skoda's L^2 estimates (for the division problem for holomorphic L^2 sections of certain holomorphic Hermitian vector bundles).

Topic 2

Title: Invariance of the plurigenera of projective manifolds of general type under holomorphic projective deformations

Supervisor: Dan Popovici

Description: This topic needs all the material covered by the above Topic 1 as a prerequisite. It is aimed at particularly motivated students wishing to reach a deep level of understanding of key aspects of the L^2 theory and of the theory of deformations of complex structures.

The main goal is to study in depth the proof given by Y.-T. Siu in [Siu98] to the following long-conjectured result:

Let $\pi : \mathcal{X} \longrightarrow \Delta$ *be a holomorphic* **projective** *family of compact complex manifolds* $X_t := \pi^{-1}(t)$ *of general type over the open unit disc* $\Delta \subset \mathbb{C}$*. Then, for every positive integer* m*, the* m*-th plurigenus* $p_m := dim_{\mathbb{C}} H^0(X_t, mK_{X_t})$ *of* X_t *is independent of* $t \in \Delta$*.*

One denotes by K_Y the canonical bundle of a complex manifold Y. By the family $\pi : \mathcal{X} \longrightarrow \Delta$ being *projective* one means that there exists a positive holomorphic line bundle on its total space $\mathcal{X}$. A compact complex manifold X is said to be of general type if its canonical bundle K_X is *big* (a positivity assumption).

Siu went on to prove in [Siu02] the above result without the *general-type* assumption on the fibres X_t of the family, but work on this dissertation will focus on the general-type case which differs in one key respect from the general case.

A key ingredient in the proof is the Ohsawa-Takegoshi L^2 extension theorem of [OT87]: given a complete Kähler manifold (X, ω), a complex submanifold $Y \subset X$ and a holomorphic section f on Y of an appropriately positive holomorphic line bundle $L \longrightarrow X$ such that f satisfies a certain L^2 condition, there exists a holomorphic extension F of f to X whose L^2-norm is bounded above by a uniform constant multiple of the L^2-norm on Y of the original f. The philosophy and the techniques of the proof are well explained in [Siu96].

Besides the Ohsawa-Takegoshi L^2 extension theorem, the proof of [Siu98] uses the theory of multiplier ideal sheaves of Nadel, whose relevant details are well explained in [Dem00].

Prerequisites (for both topics): basic notions in several complex variables (holomorphic and (pluri)subharmonic functions, currents, elliptic differential operators) and in complex geometry (complex manifold, complex structure, Hermitian metrics, connections on and curvature of holomorphic vector bundles, basic notions of Hodge Theory).

References (for both topics).

[Dem97] J.-P. Demailly — *Complex Analytic and Algebraic Geometry*—http://www-fourier.ujf-grenoble.fr/ demailly/books.html

[Dem00] J.-P. Demailly — *Multiplier Ideal Sheaves and Analytic Methods in Algebraic Geometry* — Lectures given at the ICTP School "Vanishing Theorems and Effective Results in Algebraic Geometry" held in Trieste, Italy, April 24 – May 12, 2000.

[OT87] T. Ohsawa, K. Takegoshi — *On The Extension of L^2 Holomorphic Functions* — Math. Z. **195** (1987) 197-204.

[Siu96] Y.-T. Siu — *The Fujita Conjecture and the Extension Theorem of Ohsawa-Takegoshi* — in *Geometric Complex Analysis* ed. Junjiro Noguchi *et al*, World Scientific Publishing Co. 1996, pp. 577 - 592.

[Siu98] Y.-T. Siu — *Invariance of Plurigenera* — Invent. Math. **134** (1998), 661-673.

[Siu02] Y.-T. Siu — *Extension of Twisted Pluricanonical Sections with Plurisubharmonic Weight and Invariance of Semipositively Twisted Plurigenera for Manifolds Not Necessarily of General Type* — in *Complex Geometry* (Göttingen, 2000), 223-277, Springer, Berlin, 2002.

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Further details: I will be available from March to July 2026 to supervise an M2 student working on one of these topics. Moreover, work on this dissertation can be a springboard for future PhD work that I will be glad to supervise.