

Feuille 3

Simplicité de A_n ($n \geq 5$)

1. M.g. $n \geq 3 \Rightarrow A_n = \langle (abc) \rangle_{a < b < c \leq n}$

2. M.g. $n \geq 3 \Rightarrow A_n = \langle (12a) \rangle_{a=3, \dots, n}$

Indication:

$1 \rightarrow 2 \rightarrow b$	$2 \rightarrow 1 \rightarrow b$	$1 \rightarrow a \rightarrow c$
$\downarrow \tau$	$\downarrow \tau$	$\downarrow \tau$
$2 \rightarrow a \rightarrow b$	$1 \rightarrow a \rightarrow b$	$a \rightarrow b \rightarrow c$

$\downarrow \tau = (12a)$ $\downarrow \tau = (21a)$ $\downarrow \tau = (1ab)$

3. M.g. $n \geq 5 \Rightarrow (123) \sim_{A_n} (abc)$

Indication: $(abc) = \tau (123) \tau^{-1} \Rightarrow (abc) = \tau (45) (123) (45) \tau^{-1}$

4. M.g. $n \geq 4 \Rightarrow (ab)(cd) \sim_{A_n} (12)(34)$

Indication $(ab)(cd) = \tau (12)(34) \tau^{-1} \Rightarrow (ab)(cd) = \tau (12)(12)(34)(12) \tau^{-1}$

5. M.g. A_5 est simple.

Indication:

1) Si $(abcde) \in H \triangleleft A_5$, alors considérer $((ab)(cd)(abcde))^2$

2) Si $(ab)(cd) \in H \triangleleft A_5$, alors considérer $(de)(ac)(cd)(ab)$

6. M.g. A_n est simple si $n \geq 5$

Indication:

1) Si $\sigma = (abcd \dots) \sigma_2 \dots \sigma_k \in H \triangleleft A_n$, considérer $[(abc), \sigma]$,

2) Si $\sigma = (ab)(cd) \sigma_3 \dots \sigma_k \in H \triangleleft A_n$, considérer $[(abc), \sigma]$,

3) Si $\sigma = (abc)(def) \sigma_3 \dots \sigma_k \in H \triangleleft A_n$, considérer $[(bcd), \sigma]$.