M2 Internship

Eigenframe singularities of minimizing matrix fields, applications to liquid crystals

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Given a field of symmetric matrices $M(x)$ depending smoothly on $x \in \Omega \subset \mathbb{R}^n$, it is in general not possible to diagonalize it smoothly, i.e., to find an eigenframe which depends smoothly on x. Discontinuities of the eigenframe occur when the eigenvalues change multiplicity. This type of singularity plays a fundamental role in the modelization of liquid crystals. In that case, the matrix field is not only smooth, but it minimizes an energy, and solves a nonlinear elliptic PDE of the form $\Delta M = f(M, \nabla M)$. Then it becomes possible to say something more precise about the singular set of the eigenframe. The first goal of the internship will be to understand the recent work [2] where the authors show, in a precise context involving 3×3 matrix fields on $\Omega \subset \mathbb{R}^3$ which solve a harmonic map problem, that the singular set has a specific geometric structure. The next goal will be to explore possible strengthenings of this result: to more precise estimates using the methods of [1, 3], and to more general matrix fields and energies.

References

[1] Cheeger, J., Naber, A., and Valtorta, D. Critical sets of elliptic equations. *Commun. Pure Appl. Math. 68*, 2 (2015), 173–209.

[2] Geng, Z., and Wang, C. Eigenframe discontinuities of the Q-tensor model. *Calc. Var. Partial Differ. Equ. 64*, 9 (2025), 19. Id/No 277.

[3] Naber, A., and Valtorta, D. Volume estimates on the critical sets of solutions to elliptic PDEs. *Commun. Pure Appl. Math. 70*, 10 (2017), 1835–1897.