Nom de l'UE	UE Obligatoire ou Optionnelle	ECTS	Type	H CM	H CTDI	H TD	H TD LV	H TP	H TPDE	H TPT
Mise à niveau	F	**0**		0		24	0	0	0	0
Algorithmique et calcul scientifique	O	**3**		12		8		16		
Science des données	O	**3**		12		8		16		
Equations aux dérivées partielles	O	**6**		24		24		12		
Optimisation (MAPI3)	O	**6**		24		24		12		
Probabilités (MAPI3)	O	**6**		24		24		12		
Simulation aléatoire (MAPI3)	O	**6**		18		20		24		
Découverte du monde de l'entreprise	F	**1**				10				
LANGUE VIVANTE	O	**3**		0	0	0	24	0	0	0
Stage facultatif	F	**3**								
Méthodes numériques pour les EDP	O	**6**		18		24		20		
Stage en entreprise	O	**9**								
Statistiques (MAPI3)	O	**6**		24		24		12		
Image-Signal-Simulation (MAPI3)	O	**6**		18		20		24		

Algorithmic and scientific calculus 1/Algorithmique et calcul scientifique 1,
3ECTS, 12h de CM, 8 h de TD, 16h de TP, Semestre 1

LEARNING OBJECTIVES

This module aims to provide training in modern techniques for solving linear algebra problems that arise naturally in many areas of mathematical engineering. In particular, the effective resolution of large sparse systems of linear equations will be studied.

COURSE CONTENT

1) Resolution of linear systems with direct methods (4h CM, 2h TD, 4h TP)
Review of standard factorization techniques : LU, QR, Cholevsky, Band matrices

2) Sparse linear systems (4h CM, 3h TD, 4h TP)
Storage of sparse matrices : CSR, CSR, profil, diagonal, band.
Standard linear algebra operations for sparse matrices : efficient addition, multiplication
Iterative methods for linear systems solving :
- stationary iterative methods : Jacobi, Gauss Seidel, SOR
- Krylov methods : conjugate gradient, GMRES, ...

3) Eigenvalue problem (4h CM, 3h TD, 4h TP)
- iterated power methods, Rayleigh quotient, direct and inverse method, QR
- Arnoldi

Lab with Python

PREREQUISITES
Basic knowledges of linear algebra, Python.

KEYWORDS
resolution of linear system, eigenvalues, sparse matrices, Krylov methods

BIBLIOGRAPHY

- L. Amodei and JP Dedieu : Analyse numérique matricielle, Dunod, 2008
- Y. Saad : Iterative methods for sparse linear systems (2nd edition), SIAM
- Y. Saad : Numerical Methods for Large Eigenvalue Problems (2nd Edition), SIAM

Syllabus Data Science
version anglaise

Module Name :
- Data Science

Objectives and learning outcomes : (800 caractères)
- This course is intended to introduce the concepts of statistical learning and to present classical methods in regression and classification.

Course content : (1200 caractères)
- Introduction to supervised learning : sample, regression and classification, population risk, empirical risk, cross validation
- Unsupervised classification : K-mean
- k-nearest-neighbor classifiers
- Principal Component Analysis
- Linear regression, Ridge regression, Lasso regression
- Decision tree and random forest for classification and regression
- Introduction to neural networks

Bibliography : (320 caractères)

- The elements of Statistical Learning : data mining, Inference and prediction, T. Hastie, R. Tibshirani, J. Friedman

Key words : (160 caractères)
Machine learning, Statistical Learning

Assessment methodology : (240 caractères)
- The assessment is based on one examination and the evaluation of the hands-on sessions.

Partial differential equations/Equations aux dérivées partielles, 6 ECTS
24h de cours, 24h de TD et 12h de TP, Semestre 1

LEARNING OBJECTIVES

- Stationary PDEs
- Recognize a second order linear elliptic equation.
- Know how to
    * write its variational formulation.
    * show existence and uniqueness with the Lax-Milgram theorem.
    * approximate the problem with finite difference or finite element methods
- Labs in Python

COURSE CONTENT (1200 CHARACTERS)

1) Bases of functional analysis without proofs, 2 weeks
Brief introduction to distributions : definition, derivative (jump formula), convergence in D', simple examples (Dirac, Heaviside, embedding of L1 into D')
Definition of Sobolev spaces Hm of Rn or Omega open bounded subset of Rn, without Fourier.
Statement of basic properties: Hilbert, density of regular functions, embedding of H1 into C in one dimension, counter-example in dimension 2.
Domain regularities, traces and Green's formulas without proofs, Poincaré inequalities.
Statement of the Lax-Milgram theorem.

2) Existence and uniqueness for elliptic problems, 2 weeks
Variational formulations, existence-uniqueness using Lax-Milgram, continuity with respect to data, regularity and principle of maximum for the Laplacian with homogeneous Dirichlet conditions.
Other boundary conditions (TD or TP) : non-homogeneous Dirichlet, Neumann, Robin.

3) Approximation by Finite difference methods, 3 weeks
- General principle in dimension 1 or dimension 2 on rectangle
- Definition of consistency, stability, convergence, order.
- Proof on examples in one dimension

4) Approximation by Finite element methods, 5 weeks
- Finite elements. 1-D examples. General principles of variational approximation,
- 2-D or 3-D conformity of approximation spaces. Mesher, 2-D assembly.
- Cea lemma. Convergence of the method, statement and evidence in 1-D.

PREREQUISITES
ODE, Hilbertian analysis, Lebesgue integration

KEYWORDS
Elliptic PDEs, Lax-Milgram, variationnal formulation, finite element or finite diffrence approximations.

BIBLIOGRAPHY

- H. Brezis, Analyse fonctionnelle, Théorie et applications, Masson, 1983

OPTIMIZATION

LEARNING OBJECTIVES

Optimization plays a fundamental role in the design, production and management of goods and services. The fields of application of optimization are extremely varied. We can cite as examples: the shape of an object, the efficiency of a device, the functioning of an engine, the control of airspace, the choice of economic investments, etc. The aim of this course is to introduce the main optimization algorithms (without or with constraints) based on the mathematical study of optimality conditions. The methods presented in the course will be illustrated by tutorials in Python.

COURSE CONTENT

Introdcution and basic definitions

General structure of an optimization problem. Examples. Constraints of inequality and equality types. Classification of optimization problems. Denition of local and global minima. Structure.

Convex analysis

Convex sets and functions, relative interiors, separation theorems, domains and epigraphs, directional derivatives, lower semicontinuity, sugradients and subdifferentials, convex conjugacy, Fenchel duality.

Optimality conditions

Existence results, constrained and unconstrained extrema, necessary optimality conditions, sufficiency, Fermat principle, KKT conditions, the rôle of convexity, saddle points and Lagrange duality.

Algorithms

- Unconstrained optimization: descent algorithms, gradient method, line search, Wolfe conditions, the Newton algorithm, quasi-Newton algorithm (in particular BFGS algorithm).
- Constrained optimization: projected gradient, dual methods, the Uzawa algorithm, augmented Lagrangians.

PREREQUISITES

Differential calculus: gradients, Hessian matrices, Taylor development.

KEYWORDS

Optimality conditions, gradient method, optimization algorithm, convergence analysis, Lagrange and Fenchel duality.

BIBLIOGRAPHY

- |M. Bierlaire, Introduction a l'optimisation dierentiable.
- |J. Nocedal & S. J. Wright, Numerical optimization.
- |D. G. Luenberger & Y. Ye, Linear and nonlinear programming.

Syllabus Probability
version anglaise

Module Name :
- Probability

Objectives and learning outcomes :
The study of complex phenomenon from biology, economics, industry, and physics require more and more frequently the use of a family of random variables indexed by the time, or more precisely, stochastic processes.
The principal objective of this class is to introduce and to study, from examples, stochastic processes :
- martingales, i.e. the basic dynamic in dependent case ;
- Markov chains with finite or countable state space, i.e. the usual Markov dynamic, which will be useful in several models and algorithms ;
- Poisson process, i.e. the Markov dynamic in continuous time.
- Numerous models and processes will be implemented during pratical sessions (Travaux Pratiques) on Scilab, Matlab, R or Python.

Course content :
6 ECTS, 24h CM, 24h TD, 12h TP
- Conditioning
- Conditional expectation in L2.
- Markov chains
  Markov property. Classification of states. Invariant measures for finite state space, convergences. Ergodicity (LGN and TLS) without proofs.
  Examples : Wright-Fisher model, random walks on Z, Z2, Galton-Watson process…
- Poisson process, Bernoulli process. Homogeneous Poisson process.
- Introduction to Martingales
- Surmartingales, sousmartingales. Stopping theorems. Example gambler’s ruine.

Prerequisite :
- Basic knowledge in measure theory, integration, probability theory, limit theorems.

Bibliography :

- P. Barbe, M. Ledoux Probabilité, EDP sciences
- B. Bercu, D. Chafaï Modélisation stochastique et simulation : cours et applications, Dunod
- D. Foata, A. Fuchs Processus stochastiques, Processus de Poisson, chaînes de Markov et martingales, Dunod
- https://www.python.org/

Key words :
- Stochastic processes, martingales, Markov chains

Assessment methodology :
- CCTP+CP+CT

Syllabus Simulation of randomness
version anglaise

6 ECTS, 18 h CM, 20h TD, 24h TP, 12.5h projet

Module Name :

- Simulation of randomness

Objectives and learning outcomes :

The principal objective is to study some methods of simulation of random variables, and to illustrate by simulations some models and results of probability and statistics. The various modelisations and methods presented will be illustrated during pratical sessions (Travaux Pratiques) on R, Scilab, Matlab ou Python.

Course content :

- Simulation of real random variables and Monte-Carlo method
  Method of the cumulative distribution function. Rejection sampling.
  Monte Carlo methods. Central limit theorem. Error analysis.
  (optional : quantification of a probability on R)
- Boostrap
  Jacknife, bootstrap, cross-validation
  Method of importance sampling
- Concentration inequality and/or large deviations (optional)
  Use of Chebyshev, Cramer-Chernoff, Bahadur-Rao, or Hoeffding
- Simulation and forecasting in a Gaussian model
  Gaussian vector. Simulations of a Gaussian vector (Box-Muller, linear transform). Gaussian conditioning.
  Forecasting in Gaussian linear regression. Kalman filter
  Multidimensional central limit theorem.
- Markov chain (theory done during the course of probability, same semester)
  Simulation of Markov chain (finite or continuous state space)
  Estimation of transition probabilities
  Illustration of convergences
- Introduction to MCMC
  Metropolis method
  Monte Carlo method using Markov chain. Simulated annealing
  Some stochastic algorithms : mean, median, minimisation of a cost function

Prerequisite :

- Basic knowledge in integration and probability, linear algebra.

Bibliography :

- Barbe, Ledoux (1998) « Probabilité »
- Bercu, Chafai (2007) Modélisation stochastique et simulation
- Barbe, Bertail (1995) « The weighted bootstrap ».
- https://www.python.org/

Key words :

- Simulations, Monte-Carlo, stochastic algorithms

Assessment methodology :

- CCTP+CP+CT

Syllabus- World of industry
English version

Module Name :

Discovering the business world

Objectives and learning outcomes : (800 caractères)

The course provides notions of project management. It also provides a basic understanding of resume writing. Finally, various speakers from the professional world come to present mathematical problems that their company encounters and solves.

Course content : (1200 caractères)

All the notions will be illustrated by feedback from experience from the industrial world.

The place of applied mathematics in the major functions of the company will be reviewed. Case studies are provided. At the end of the course, the student will have acquired the main fundamentals of organizations and project management in companies. It will have enriched its vocabulary in the field in order to be able to decode internship or job announcements, prepare for professional interviews and take its first steps in the company. Detailed content of the course :

- The different types of companies
- The different organizations
- The main functions of the company
- Standard tools for flow, information and resource management
- Mathematics in business
- Project Management
- Culture, the stakes
- Feedback from experience
- Case Studies

Key words : (160 caractères)

- Project management, company, resume

Assessment methodology : (240 caractères)

- none

Syllabus English
version anglaise

Name of module :
- English

Objectives and learning outcomes :

Level C1 of the CEFRL (Common European Framework of Reference for Languages)

- Develop the skills necessary to students to succeed in their future professional lives in a variety of cultural contexts.

- Acquire the necessary linguistic autonomy and improve the specialized language allowing professional integration and communication of scientific expertise in an international context.

Competences

Speak fluently in front of an audience, using registers adapted to different contexts and different interlocutors.
Use a foreign language other than French easily: written and oral comprehension and expression:
- Understand a scientific or professional article written in English on a subject related to their field.
-Interact orally in English in order to succeed in formal and informal exchanges during lectures, meetings or professional interviews.

- Defend your application in writing (CV) or orally (job interview) in English

Course Content
- Develop skills related to the understanding of scientific or professional publications written in English as well as skills necessary to understand oral scientific communications.
- Acquire expression tools in order to master an oral and/or written presentation and to tackle a critical discussion in the scientific field, (e.g. rhetoric, linguistic elements, pronunciation...) .
- Acquire the linguistic and cultural elements of professional communication (CV, cover letter, interview)

Prerequisite : Level B2 CEFRL

Key words
Project - Scientific English - Writing - Publishing - Communications - interculturality - professionalisation

Assessment methodology : Continuous assessment:

Numerical methods for PDEs/Méthodes numériques pour les EDPs, 6 ECTS
18h de cours, 24h de TD, 20h de TP, 12.5h de projet, Semestre 2
Mutualisation avec M1ESR, les élèves du M1ESR pourraient être dispensés de la partie projet

LEARNING OBJECTIVES

Non stationary linear PDEs
- Classification of the partial differential equations and study of their basic qualitative properties of each type of equations
- Classical finite volumes or finite differences schemes for linear scalar equations.
- Numerical analysis of these schemes: stability, consistency and convergence

At the end of this teaching, the student will know how to
- recognize the nature of a PDE,
- show the well-posedness of a problem
- design an algorithm to approach the solution
- implement this algorithm and will also have a critical eye on the results obtained.

Labs in Python

COURSE CONTENT

1) Heat equation, 5 weeks
- introduction with complex systems. Ex : Navier-Stokes
- Resolution with Fourier in multi-D
- Approximation by Finite difference or finite volumes.

2) Transport equation, 5 weeks
- Introduction with complex systems (ex : Euler)
- Resolution of the linear transport equation, with the caracteristic curves, problem of boundary conditions (specific of hyperbolic problems)
- Approximation by finite difference or finite volume methods,
only in 1-D, explicit and implicit schemes,
stability Linf and L2, consistency, convergence.

3) Wave equation. 2 weeks
- Introduction with complex systems (ex : Helmotz, Maxwell)
- Approximation by finite difference or finite volume methods,

PREREQUISITES
PDE course of the 1st semester, ODE.

KEYWORDS **P**arabolic, hyperbolic, linear partial differential equations.
Finite differences or finite volumes schemes, stability, consistency, convergence

BIBLIOGRAPHY

- L. C. Evans, Partial Differential Equations, American Math. Soc., 1999.
- G. Allaire, Analyse numérique et optimisation: une introduction à la modélisation mathématique et à la simulation numérique, Palaiseau : Éditions de l'École Polytechnique - DL 2005
- P.A. Raviart and JM Thomas. Introduction à l'analyse numérique des équations aux dérivées partielles.

IMAGE, SIGNAL, SIMULATIONS

LEARNING OBJECTIVES

The course presents an overview of the main image processing problems: restoration, segmentation, repackaging. We will also see the main models aiming at solving them, as well as the numerical strategies allowing to calculate a solution. The methods presented in the course will be illustrated by tutorials in Python.

COURSE CONTENT

Introduction and reminders

- Reminders on imaging: convolution, windowing, sampling.
- Optimization tools for imaging: gradient and proximal gradient algorithm.
- Inverse problems and ill-posedness.

Image restoration

- Introduction to image restoration: Bayes law and MAP estimators in imaging.
- The total variation in the continuous and discrete domains.
- Image denoising, operator inversion (deconvolution, inpainting, zoom), dequantification of images, restoration of compressed images.

Sparse representations in a dictionary of atoms

- Minimization of the L0-norm, non-linear approximation, compressed sampling (case of the L0-minimization), numerical resolution in the orthonormal case.
- Minimization of the L1-norm, the compressed sampling (case of the L1-minimization), numerical resolution by the proximal gradient algorithm.
- Example of a greedy algorithm: OMP.
-

Image segmentation

- Stochastic model of a form: perimeter of a discrete form.
- Object models: estimation of the laws of colors or characteristics of an object.
- The models of Mumford-Shah, Chan-Vese and Boykov-Jolly.

PREREQUISITES

Basic Fourier analysis, optimization, differential calculus, basics in Statistics.

KEYWORDS

Imaging, deblurring, ill-posed problems, L1-minimization, total variation, compressive sensing, image segmentation, shape optimization.

BIBLIOGRAPHY

O. Scherzer, M. Grasmair, H Grossauer, M. Haltmeier & F. Lenzen, *Variational Methods in Imaging*. Vol. 167. Springer Science & Business Media, 2008.

Syllabus Internship
English version

Module Name :
Internship

Objectives and learning outcomes : (800 caractères)
The internship is for a minimum of three months between the beginning of May and the end of August. Its objective is to give a first professional experience to students in a field related to their studies in the Master MApI3.

Assessment methodology : (240 caractères)

The report

A short report, produced using Word or LaTeX, must be returned to the training managers at the end of the intership. It must be sufficiently detailed to enable the jury to assess the work without being too long. A standard length is about 30 pages (excluding annexes). The report must quickly describe the "department" in which the internship took place, describe the problems, the objectives, the main stages of the work, and finally indicate the obtained results (while specifying the methodology).

The defense

An internship defense takes place at the beginning of September. Each student has 30 minutes: 20 minutes of presentation, 5 minutes of questions and comments, 5 minutes of deliberation. A "Power-Point" or "Latex Beamer" type support will be used, the jury having a computer and a video-projector at the intern's disposal.

The jury and the mark

The jury is made up of at least two teachers and the head of the internship company who is cordially invited to participate in your defense. The grade takes into account the work done (quality, quantity and difficulty), the quality of the internship report and the quality of the presentation (quality of the support and quality of the oral expression).