M2 INTERNSHIP PROPOSAL: AN INTRODUCTION TO SEMICLASSICAL ANALYSIS OF WAVE PROPAGATION

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This internship is designed as an introduction to semiclassical analysis of wave propagation problems. More precisely, we will be interested in the Helmholtz equation outside an obstacle $\mathcal{O}$,

$$-\Delta u - k^2 u = f, \qquad \text{in } \Omega := \mathbb{R}^d \backslash \mathcal{O},$$

one of the simplest wave models, which can be obtained as the Fourier transform in time of the wave equation, and where $k > 0$ represents the frequency. In the high-frequency limit $k \gg 1$, the intuition is that a wave propagates along the rays of geometrical optics. Tools from semiclassical analysis, which allow to localise both in position and direction (that is, microlocally) in the regime $k \to \infty$, permit in particular to give a rigorous meaning to this heuristic. One of the appeals of this field is that it allows to understand the behaviour of solutions to PDE thanks to geometrical intuitions.

The first stage of this internship will be to read the research paper [1], which proves elegantly a central result in this thematic: when all the rays of geometrical optics escape to infinity, Helmholtz solutions verify the same resolvent estimates, which quantify the decay of a wave, as in the free case. Then, if time allows, the intern will try to show a first research result by using the techniques from [1] in another context: for example, in situations where the obstacle is not in $\mathbb{R}^d$ anymore but in a bounded domain with an absorbing layer. Such a result would have interesting applications to numerical analysis, where truncating an unbounded domain is a common strategy.

This internship could potentially lead to a Ph.D thesis, on themes such as applications of semiclassical analysis to numerical analysis, theoretical analysis of nonlinear waves, or control of wave and Schrödinger equations.

References

[1] N. Burq. Semi-classical estimates for the resolvent in nontrapping geometries. *Int. Math. Res. Not.*, 2002(5):221–241, 2002.

[2] M. Zworski. *Semiclassical analysis*, volume 138 of *Grad. Stud. Math.* Providence, RI: American Mathematical Society (AMS), 2012.

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