# Perturbation methods of the dependent laws of the entries of a simulation code

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## 1 Industrial context

The company EDF currently exploits 56 nuclear reactors in France of which the activity is submitted to various safety restrictions guaranteeing the absence of risks for the population and the environment, including in case of an incident or accident. In particular EDF must prove the absence of risk concerning the integrity of the fuel composing the heart of the reactor in case of a LOCA (loss of coolant accident).

This safety demonstration is based on the CATHARE code simulating fluid flows and thermal exchanges inside the primary circuit of the reactor in case of such an event. This numerical simulation code, which we denote $G$ in what follows, allows to model the different components of the reactor and their functioning in a given scenario. It is based on a set of differential equations controlling the dynamics of the primary fluid during the simulated scenario.

The equations implemented in CATHARE depend on different physical parameters noted $X_1, \ldots, X_d$ whose value, determined experientally, is affected by uncertainties modelled by some probability laws $f_i$ for each $X_i$. The variables $X_i$, considered as the inputs of the code $G$, induce an uncertainty on the output $Y = G(X_1, \ldots, X_d)$ that one characterises in the framework of the safety study by a quantile of the laws of $Y$ denoted $q^\alpha(Y)$.

## 2 Robustness analysis and perturbation of laws method

The robustness analysis is an emerging approach in the domain of uncertainty quantification and aims at quantifying the impact on a quantity of interest in the output of the code $G$ of the uncertainty associated to the characterisation of the laws of the uncertain inputs of the code $G$. The description of the physical behavior of the fluid is potentially crucial in the safety demonstration, so EDF R&D has developed a method allowing to analyse the robustness of the calculation of the quantile $q^\alpha(Y)$ in terms of the characterisation of the laws $f_i$ themselves.

This method consists in perturbing (modifying) the laws $f_i$ for $i = 1, \ldots d$ and depends therefore on a specific technique for perturbing the laws based on Fisher information [3]. This technique supposes that the law $f_i$, for a given $i$, is contained in a parametric family $\mathcal{P}_i = \{f_{i\theta}\}_{\theta \in \Theta}$. A perturbation of level $\delta > 0$ of $f_i$ corresponds then to an element of

$$\Lambda_{i\delta} := \{f \in \mathcal{P}_i \mid d_i(f, f_i) = \delta\},$$

the sphere centered at $f_i$ and of radius $\delta$ for the Fisher-Rao distance $d_i$ on $\mathcal{P}_i$. This distance is defined as the geodesic distance associated to the Fisher information metric of $\mathcal{P}_i$, which endows the family $\mathcal{P}_i$ of the structure of a riemannian manifold [1, 2].

## 3 Goal of the internship

This recently proposed perturbation method is used in a methodological framework of which one of the principal hypotheses is the independence of the random variables $X_i$, inputs of the code $G$, and consisting in perturbing each input separately. In practice, though, the independence hypothesis of the $X_i$ cannot be systematically justified and some of these entries may be effectively correlated. Furthermore one would like to study robustness with respect to a joint perturbation of multiple code inputs. In both cases, an extension of the current method is required allowing one to perturb the joint law of the multiple entries of the code $G$. This internship's goal is to develop such a method by taking into account a possible dependence structure between the different variables $(X_1, \dots, X_d)$.

An idea to explore is based on the theoretical representation of multivaried laws by copulas. One can illustrate it as follows in the bidimensional case which will be studied in a first time.

One supposes that the code $G$ has only 2 entries $X_1, X_2$ whose uncertainty is modeled respectively by the families of laws:

$$\mathcal{P} = \{p_\theta\}_{\theta \in \Theta} \quad \text{and} \quad \mathcal{Q} = \{q_\xi\}_{\xi \in \Xi},$$

admitting a density on $\mathbb{R}$. One supposes furthermore that the dependece structure between $X_1$ and $X_2$ is modeled by a family of copulas $\{C_\rho\}_{\rho \in \varrho}$ admitting a density $\{c_\rho\}_{\rho \in \varrho}$ on $[0,1] \times [0,1]$. We wish to construct the family $\mathcal{R}_\rho$, with $\rho \in \varrho$ fixed, of the following densities on $\mathbb{R}^2$:

$$f_{(\theta,\xi,\rho)}(x_1, x_2) = c_\rho\big(F_{p_\theta}(x_1), F_{q_\xi}(x_2)\big) p_\theta(x_1) q_\xi(x_2),$$

where $F_{p_\theta}(x_1)$ and $F_{q_\xi}(x_2)$ correspond respectively to the repartition functions of $p_\theta$ and $q_\xi$. For this one can first compute for fixed $\rho$, the Fisher information metric $g^\rho$ for this new family $\mathcal{R}_\rho := \{f_{(\theta,\xi,\rho)}\}_{(\theta,\xi) \in \Theta \times \Xi}$ and the Fisher-Rao distance in a toy case where $\mathcal{P}$ and $\mathcal{Q}$ are gaussians and $C_\rho$ is a copula belonging to an explicit parametric family (usually an elliptic family).

The expected result is then the definition and the implementation of a methodology of robustness analysis on this kind of case study, allowing the possibility of varying the parameter $\rho$, and of modifying the copulas.

**Profile of the candidate**

- Student in a M2 in mathematics or in last year of an engineering school with specialisation in applied maths.
- Solid knowledge in statistics and probability.
- Good coding skill in a specific scientific programming language (Python, R, Matlab,...).
- If possible, basic knowledge of numerical methods and differential geometry.
- This internship is addressed to students having a taste for mathematics and interest in research.

**Key words**: robustness analysis, geometry of information, copulas.

## References


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[2] Manfredo Perdigao Do Carmo and J Flaherty Francis. *Riemannian geometry*, volume 2. Springer, 1992.

[3] Clement Gauchy, Jerome Stenger, Roman Sueur, and Bertrand Iooss. An information geometry approach to robustness analysis for the uncertainty quantification of computer codes. *Technometrics*, 64(1):80–91, 2022.