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PROPOSITION DE STAGE EN COURS D'ETUDES

Référence : **DTIS-2026-02** *(à rappeler dans toute correspondance)*	Lieu : Toulouse
Département/Dir./Serv. : DTIS/RIME	Tél. : 05.62.25.28.11
Responsable(s) du stage : Julien Demange-Chryst, Jérôme Morio	Email. : julien.demange-chryst@onera.fr jerome.morio@onera.fr

DESCRIPTION OF THE INTERNSHIP

Thématique(s) : Applied mathematics and their interactions, scientific computing

Type de stage : ☒ Fin d'études bac+5 ☒ Master 2 ☐ Bac+2 à bac+4 ☐ Autres

Title : Conformal prediction for rare event estimation by active learning

Description : Many physical systems are schematically described by a relation of the form Y = ϕ(**X**), where the multidimensional input **X** is assumed to be random and the output Y is determined via the deterministic function ϕ. A common example is the analysis of a black box numerical code: ϕ represents a calculation code and **X** represents the external conditions in which this calculation is performed. One example is a finite element code, whose complexity makes it impossible to perform any analytical study of the function ϕ and therefore of the output Y.

The failure of critical systems can have potentially dramatic environmental, economic or human consequences. Fortunately, such events are generally rare. However, for safety and certification reasons, it is crucial to quantify the risk associated with such systems: it is the reliability analysis. Mathematically speaking, the failure of the system is modeled as an event of the form {Y>t}, where t is a failure threshold defined by physical considerations. Then, the risk associated with the system is quantified by the failure probability p_t = P(Y>t). Nevertheless, because of the complexity of the numerical code ϕ, p_t cannot be calculated analytically.

Estimating the failure probability is now a widely studied research topic in the literature (crude Monte Carlo, importance sampling [1], subset simulation [2]). Furthermore, since the numerical code ϕ can be very time-consuming to evaluate, a widely used family of estimation methods consists of replacing ϕ with a surrogate model, such as a Gaussian process (AK-MCS algorithm [3]). However, the approximation error of the meta-model is difficult to control, and these methods do not allow the estimation error of p_t to be quantified correctly.

In this context, the objective of this internship is to build a valid confidence interval for the failure probability, when it is estimated using a surrogate model, which takes into account the approximation error of ϕ. To do so, one interesting avenue of research is conformal prediction [4]. This is a statistical method for quantifying the uncertainty of any prediction model by constructing statistically valid prediction sets. This research topic is currently extremely dynamic in the literature.

After reviewing the literature on reliability analysis, surrogate modeling and conformal prediction methods, the first area of work for the internship would be to apply a conformal prediction approach to the meta model used, based on recent work [5,6], and to propagate it to the estimation of the failure probability. A second area of work would be to take into account the adaptive nature of reliability algorithms, such as the AK-MCS algorithm. Indeed, active learning of the meta-model no longer guarantees the fundamental assumption of data exchangeability. Conformal prediction methods adapted to this situation will need to be explored [7] to guarantee the statistical validity of the confidence interval constructed.

The method proposed during the internship will be applied to various aerospace test cases.

[1] J. A. Bucklew. Introduction to rare event simulation. *Springer Series in Statistics. Springer-Verlag, New York*, 2004.

[2] F. Cérou, P. Del Moral, T. Furon, and A. Guyader. Sequential Monte Carlo for Rare Event Estimation, *Statistics and Computing*, 22(3):795-808, 2012.

[3] Echard, B., Gayton, N., & Lemaire, M. AK-MCS: an active learning reliability method combining Kriging and Monte Carlo simulation. *Structural safety*, 33(2), 145-154, 2011.

[4] Angelopoulos, A. N., & Bates, S. ,A gentle introduction to conformal prediction and distribution-free uncertainty quantification. *arXiv preprint arXiv:2107.07511*, 2021.

[5] Jaber, E., Blot, V., Brunel, N. J., Chabridon, V., Remy, E., Iooss, B., & Leite, A. Conformal approach to gaussian process surrogate evaluation with marginal coverage guarantees. *Journal of Machine Learning for Modeling and Computing*, 6(3), 2025.

[6] Pion, A., & Vazquez, E. Gaussian process interpolation with conformal prediction: methods and comparative analysis. In *International Conference on Machine Learning, Optimization, and Data Science,* 218-228, Springer Nature Switzerland, 2024.

[7] Barber, R. F., Candes, E. J., Ramdas, A., & Tibshirani, R. J. Conformal prediction beyond exchangeability. *The Annals of Statistics*, 51(2), 816-845, 2023.

Is it possible to consider working in pairs ? No

Methods to be implemented :

☒ Theoretical research	☐ Synthesis work
☒ Applied research	☐ Documentation work
☐ Experimental research	☐ Participation in a project

Possibility of extension to a PhD : Yes

Duration of the internship : Minimum : 4 Maximum : 6

Desired period : First semester of 2026

PROFIL DU STAGIAIRE

Required knowledge and level:	Formation:
Applied mathematics, probability, statistics, proficiency in a programming language.	Third year of engineering school, Master's degree.