Master level internship

Parameter estimation for repeated quantum measurements

Tristan Benoist: tristan.benoist@math.univ-toulouse.fr
Clément Pellegrini: clement.pellegrini@math.univ-toulouse.fr

Repeated quantum measurements describe contemporary experiments in quantum technologies, in particular in quantum optics. From a mathematics point of view, the formalism of quantum mechanics is a generalization of the classical probability to non commutative objects. Typically probability vectors are replaced by positive semi-definite matrices of trace one (density matrices) and (sub-)stochastic matrices are replaced by positive maps on positive semi-definite matrices.

Many standard results on Markov chains can be generalized to repeated quantum measurements (Perron-Frobenius theorem, law of large numbers, central limit theorem ...). The main remaining open problem is the study of parameter estimation for these models. It is of course crucial to experiments as it could lead to better estimation algorithms allowing for a finer control of quantum phenomenons.

Mathematically, quantum measurements are modeled by families of positive endomorphisms $\{\Phi_a\}_{a\in\mathcal{A}}$ on positive semi-definite matrices such that their sum $\Phi = \sum_a \Phi_a$ preserves the trace : $\mathrm{tr}(\Phi(A)) = \mathrm{tr}(A)$. Given a density matrix ρ, the law of the outcomes of a sequence of measurement is given by

$$\mathbb{P}(a_1, \dots, a_n) = \mathrm{tr}(\Phi_{a_n} \circ \dots \circ \Phi_{a_1}(\rho)).$$

Then, if $\{\Phi_a\}_a$ depends on a parameter θ, denoting $\{\Phi_a^\theta\}_a$ the dependency in θ,

$$\mathbb{P}_\theta(a_1, \dots, a_n) = \mathrm{tr}(\Phi^\theta_{a_n} \circ \dots \circ \Phi^\theta_{a_1}(\rho))$$

defines a statistical model $(\theta \in \Theta, \mathbb{P}_\theta)$.

Questions like identifiability and consistency of the maximum likelihood can be solved using standard techniques borrowed either from quantum spin systems or hidden Markov models. The main open question is whether these models are asymptotically gaussian. Proving such a result would unlock access to a whole family of algorithms and bound for parameter estimation and would also allow us to better understand the geometry of these models with connections to linear response in physics.

For the interested student, the goal of the internship would be to familiarize his or herself with the model of repeated quantum measurements. Then a first step would be a translation of the proof of consistency of the maximum likelihood estimator from hidden Markov models to repeated quantum measurements. This internship can be continued into a Ph. D. thesis whose goal would be to prove asymptotic normality starting eventually with simpler models. It may be possible to find models that are not asymptotically normal but behave as different standard laws like Poisson models.

More generally, we expect that the proofs developed in this project would be useful to establish asymptotic normality for more general hidden Markov models.