Topic of the internship

Statistical Properties of a Regression Estimator on Riemannian Manifolds : Application to ground robot pose trajectory estimation.

Location : ISAE-SUPAERO, Département d'Ingénierie des Systèmes Complexes
Co-supervisors : Emilien Flayac (ISAE-SUPAERO), emilien.flayac@isae.fr, Benjamin Bobbia (ISAE-SUPAERO), benjamin.bobbia@isae.fr, Florence Nicol (ENAC) nicol@recherche.enac.fr et Stéphane Puechmorel (ENAC) stephane.puechmorel@enac.fr

General Context

The problem of estimating the trajectory of a drone is central to trajectory tracking, localization, guidance, and planning. Three-dimensional drone dynamic models, composed of translation and rotation, are generally nonlinear, which makes trajectory prediction and error propagation within the model very challenging. Moreover, sensors commonly found on drones, such as monocular cameras or lidars, are subject to disturbances and do not allow measurement of the full system state, but only, for example, distances, directions, or optical flow. Thus, trajectory estimation must generally be performed using partial and noisy measurements. Estimation approaches based on nonlinear observers provide robust deterministic methods but are rarely accompanied by quantitative analyses in the presence of noise. Bayesian filters, such as extended Kalman filters (EKF), unscented Kalman filters (UKF), or particle filters, are often limited : the former lose accuracy under strong nonlinearity, while the latter are computationally expensive.

On the other hand, this problem can also be handled using optimization-based methods such as statistical regression (parametric and non-parametric). A particular case in the context of dynamic estimation is Moving Horizon Estimation (MHE), which consists in determining a segment of the trajectory by minimizing the error between real data and model-predicted data over a fixed-size time window. These techniques can handle nonlinear dynamics and measurements and are robust to disturbances.

A fundamental issue in this field is the representation of rotations. Euler angles only provide a local description of motion, whereas representations based on differentiable manifolds and/or Lie groups, such as rotation matrices (spaces $SO(2)$ or $SE(2)$ for instance), are global and allow one to exploit well-established geometric properties from the literature, at the cost of nonlinear constraints that must be imposed on the variables during estimation. The design of geometric alternatives to standard observers and Bayesian filters, such as the Invariant Extended Kalman Filter (IEKF), is already a well-studied area. However, statistical estimation based on regression that relies on the intrinsic geometry of the ambient space, without embedding it into a Euclidean space, is an emerging and still poorly understood field.

In this internship, we will consider a mobile system whose orientation is unknown ; the objective will be to estimate this orientation using only angle measurements between the mobile and reference points called "landmarks". Accelerated gradient descent algorithms on Riemannian manifolds have already been introduced and developed for the case of noiseless angle measurements between the mobile and the landmarks. The main challenge of this internship will be to understand the effects of adding noise to the measurements—in particular, the impacts on estimation quality as well as on fundamental properties of the estimation problem, such as existence and uniqueness of solutions.

Internship Objectives

- Analysis of the statistical properties of the rotation estimator in the presence of random errors in the landmark measurements. This includes, but is not limited to, understanding the bias, variance, and asymptotic behavior (consistency and asymptotic normality).
- Study of the well-posedness of the optimization problem. This condition ensures the existence and uniqueness of the solution. In the purely deterministic setting, there exist landmark configurations that do not guarantee well-posedness. The objective is to develop a hypothesis test to verify this well-posedness and, if possible, highlight conditions on the noise that could lead to loss of existence or uniqueness of solutions for the optimization problem.
- (Bonus) Extend the statistical results obtained for accelerated Riemannian gradient descent on $SO(2)$ to $SE(3)$ by adding a translational component to the mobile's motion. This last point is a significant challenge, since unlike the $SO(2)$ case, the algorithms themselves are less well understood.

Référence

[1] P.-A. Absil, R. Mahony, and R. Sepulchre. *Optimization Algorithms on Matrix Manifolds*. Princeton University Press, Princeton, NJ, 2008.

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[3] Cédric M. Campos, David Martín de Diego, and José Torrente. Momentum-based gradient descent methods for lie groups, 2025.

ISAE SUPAERO - Institut Supérieur de l'Aéronautique et de l'Espace

10, avenue Edouard-Belin - BP 54032 - 31055 Toulouse CEDEX 4 - FRANCE

Tél : 33 (0)5 61 33 80 80 - Télécopie : 33 (0)5 61 33 83 30 - Site internet : www.isae-supaero.fr