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Iterated convolutions and the local limit theorem

Description. The aim of this Master 2 internship is to study numerical schemes for wave propagation phenomena in one or several space dimensions, and more specifically to study the asymptotic behavior of such numerical schemes for large times. In the framework of uniform Cartesian meshes, this problem can be recast as the study of iterated convolutions of a given sequence (namely, that of the coefficients that define the numerical scheme). This problem is therefore entirely analogous, for real nonnegative sequences, to the study of the large time asymptotic behavior of random walks on the network $\mathbb{Z}^d$ for which the local limit theorem predicts a Gaussian behavior. The classification of asymptotic behaviors beyond the probabilistic case is far richer. The goal of the internship is to first digest the asymptotic expansion and large time error estimates in the one-dimensional case, the final goal being to generalize this approach in several space dimensions.

The student will be strongly encouraged to implement simple, or more elaborate, numerical schemes in order to illustrate and/or predict the results proved during the internship.

Prerequisite : for this internship, it is desirable to know the theory of integration (L^p spaces, convolution...), the standard results from Fourier analysis (Fourier series and Fourier transform, Parseval-Bessel equality, Plancherel's theorem...) and some elements of complex analysis (holomorphic functions, Cauchy theorem, contour deformation...).

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Location : the student will work within the Institut de Mathématiques de Toulouse on the Université Paul Sabatier campus (Toulouse, France).

Duration : 4/5 months from March to June/July 2025.

Gratification : this internship may give rise to payment of a monthly bonus.

References : here are some references that will serve during this internship :

— J.-F. Coulombel et G. Faye, The local limit theorem for complex valued sequences: the parabolic case, *Comptes Rendus, Mathématique*, Académie des Sciences, 2024 (à paraître).
— V. Petrov, *Sums of independent random variables*, Springer-Verlag, 1975.
— E. Randles et L. Saloff-Coste, Convolution powers of complex functions on $\mathbb{Z}^d$, *Revista Matematica Iberoamericana*, Volume 33, 2017.