M2RI INTERNSHIP IN APPLIED MATHEMATICS: ON DISCRETE BRASCAMP-LIEB INEQUALITIES FOR GLOBAL SENSITIVITY ANALYSIS

ALDÉRIC JOULIN AND OLIVIER ROUSTANT

The development of Global Sensitivity Analysis (GSA) of numerical model outputs has become increasingly popular in the three last decades. It is by now an essential international research topic, combining modern mathematical and statistical tools to computer experiments, and has many consequences in engineering for industry. Recall that the principle of GSA is to quantify the influence of continuous input random variables on the output of a multivariate function $f : \mathbb{R}^d \to \mathbb{R}$ which might be expensive to evaluate since dimension d is large. These variables can represent calculation codes that model complex phenomena or artificial intelligence algorithms which are not well understood.

When it comes to quantifying influence or uncertainty, practitioners tend to use variance-based Sobol indices because of their clear interpretation in terms of ANOVA decomposition, at least under the assumption of independent entries. Despite their clear interpretability, their estimation requires numerous calculations, making them an expensive computational tool. When the derivatives of f are available, other sensitivity indices called DGSM (Derivative-based Global Sensitivity Measure) reveal to be efficient since they are cheaper to compute. As observed in [4] and further studied in detail in [5], Sobol indices and DGSM are connected by the so-called Poincaré inequality. However this functional inequality has some drawbacks. On the one hand there exists some usual probability measures which do not satisfy the usual Poincaré inequality (*e.g.*, heavy-tailed distributions) and on the other hand the DGSM-based upper bounds might be too large. To overcome this difficulty, we recently introduced in [3] an appropriate weight in the DGSM indices, leading to the notion of weighted DGSM indices. As such, they are connected to the standard Sobol indices by a weighted Poincaré inequality. This new inequality is sharp in most of the cases and provides an additional degree of freedom by choosing conveniently the weight to enhance the precision of the upper bound.

Motivated by computational reasons, the purpose of this internship is to consider a discretized version of the continuous setting emphasized in our previous work [3]. More precisely, we aim at studying the connections between GSA and functional inequalities for discrete structures, for which the input random variables are chosen to be discrete (Bernoulli, Poisson, geometric, etc.). Because of the lack of chain rule, establishing functional inequalities in discrete setting is very challenging. To that aim, we identified three tasks:

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• To extend the weighted Poincaré inequalities of [3] to the discrete setting. It seems to us that the spectral tools at the basis of the results in [3] can be adapted to this context.

• To use the intertwining approach introduced in [2] for birth-death processes to obtain more general weighted Poincaré inequalities. We have in mind a discrete analogue of the so-called generalized Brascamp-Lieb inequalities studied for diffusions in [1].

• At least in the Bernoulli case, to see how these Brascamp-Lieb inequalities are connected to the theory of influences in Boolean analysis, in which the famous $L^1 - L^2$ Talagrand variance inequality [6], which is a refined Poincaré inequality, plays a crucial role.

This internship can be seen as a project at the interface between applied statistics, probability and functional analysis. In the case of a potential financial support, such an internship might lead to a PhD Thesis prepared under our joint supervision at the Institut de Mathématiques de Toulouse.

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(A. Joulin) UMR CNRS 5219, Institut de Mathématiques de Toulouse, Université de Toulouse, France

Email address: `mailto:ajoulin(at)insa-toulouse.fr`

URL: `http://perso.math.univ-toulouse.fr/joulin/`

(O. Roustant) UMR CNRS 5219, Institut de Mathématiques de Toulouse, Université de Toulouse, France

Email address: `mailto:roustant(at)insa-toulouse.fr`

URL: `https://olivier-roustant.fr/`