Master 2 Research Internship Proposal

Laboratoire Jean Alexandre Dieudonnée in Nice

Academic Year 2025/2026

Numerical Simulation of Cellular Electrophysiology using Finite Volume Methods

Coupled Poisson-Hodgkin-Huxley Model

The main objective of this internship is the study, implementation, and analysis of a numerical scheme for simulating the electrical activity of excitable cells (neurons or cardiomyocytes). The underlying mathematical model is a coupled formulation:

1. The **Poisson** equation (or a simplified diffusion-reaction type equation) to describe the electric potential in the intra- and extracellular domains.
2. The **Hodgkin-Huxley** model (or a simplified variant) to model the time- and potential-dependent transmembrane ionic flux.

In the first stage, the student will have to familiarize himself/herself with the Poisson-Hodgkin-Huxley system and the Discrete Duality Finite Volume (DDFV) method. In a second step, the student will have to implement the 2D numerical solver starting from an existing code. The internship will focus on a mixed geometric configuration:

- The **cell membrane** is treated as a 1D interface (reduced dimension).
- The **intracellular** and **extracellular** domains are represented in 2D.

One of the major challenges is the effective coupling between these two types of domains and the handling of the discontinuity at the membrane level.

Internship Objectives

1. **Literature Review:** In-depth study of the associated numerical methods.
2. **Numerical Scheme Development:** Setting up a **DDFV** scheme for the spatial discretization of the Poisson equations in the 2D domains. The choice of the temporal scheme (e.g., Crank-Nicolson or Implicit/Explicit Euler depending on the term) will be guided by stability and accuracy considerations.
3. **Coupling and Implementation:** Management of the coupling between the Partial Differential Equations (PDEs) in the volume and the Ordinary Differential Equations (ODEs) of the Hodgkin-Huxley membrane at the interface. The implementation will be performed on a scientific computing platform (in Fortran 90).
4. **Numerical Analysis:** Conducting convergence studies and stability analysis of the developed scheme.
5. **Biological Applications:** Simulation of key phenomena such as action potential propagation and analysis of the impact of membrane parameters on cellular dynamics (neuron or cardiac application).

Required Student Profile

This subject is aimed at a Master 2 student in Applied Mathematics, Scientific Computing, or Numerical Engineering.

- **Strong Foundations in Numerical Analysis:** Proficiency in PDE discretization methods (Finite Differences, Finite Elements, or Finite Volumes).
- **Scientific Computing and Programming:** Ability to efficiently implement numerical algorithms. Experience in Fortran is a major asset.
- **Specific Knowledge Appreciated:**
  - Knowledge of **Finite Volume methods** (highly appreciated).
  - Notions of modeling in Biology/Physiology (a plus).

Internship Environment

The intern will be integrated into the PDE and Numerical Analysis team in Nice, and will benefit from regular supervision by Stella Krell (Université Côte d'Azur) and Julien Moatti (Bordeaux-INP). There is a possibility of continuing the project with a PhD.

Estimated Duration:	4 to 6 months
Internship Location:	Laboratoire Jean Alexandre Dieudonné in Nice
Contact:	Stella Krell (stella.krell@univ-cotedazur.fr) and Julien Moatti (julien.moatti@bordeaux-inp.fr)