Master thesis proposal
Testing for linear Hawkes process

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Student profile: Master 2 student in Applied Probability, Statistics. Programming skills in Python will be appreciated.

Keywords: Hawkes processes, Tests procedure, Maximum likelihood estimators

Working place: Institut de Mathématiques de Toulouse or LPSM.

Remuneration: Standard internship grant: 550 euros/month

Objective and content of the Internship This internship proposal is part of the research project "HAPPY" funded by the ANR, which deals with better understanding non-linear Hawkes processes. Hawkes processes are point processes used to model the occurrences of a given event and their impact on the probability of future events. For this reason, they became popular in many domains, for example in finance, neuroscience or epidemiology. A main challenge comes from the fact that the intensity of a Hawkes is random and depends on the whole past of the process, which makes the process non markovian in a general setting.

Linear Hawkes processes are easier to handle. A linear Hawkes process N_t admits an intensity of the form

$$\lambda_t = \mu + \int_0^t h(t-s)dN_s$$

where $\mu > 0$ and h is a L^1 function. In this internship we want to develop testing procedures to assess that observations derive from a linear Hawkes process. The question of constructing a non parametric test (for any μ and h with $||h||_1 < 1$) is difficult as it requires the construction of estimators for μ and h. Therefore, we will first focus on a parametric setting where h is an exponential function:

$$\lambda_t^{(\mu,\alpha)} = \mu + \int_0^t \exp(-\alpha(t-s))dN_s \quad \mu, \alpha > 0$$

This case is interesting on its own as then, the Hawkes process is a Markov process. Such modelisation is often encountered in practice therefore having test procedure that permits to validate their use is important.

More precisely, we assume that we observe a sequence of times $(T_1 \leq T_2 \leq ...)$ on a long time interval generated from a point process N. We want to test the null hypothesis that

$$H_0 = \{\exists \mu, \alpha > 0 \text{ such that intensity of the counting process } N_t \text{ is } \lambda^{(\mu,\alpha)}\}.$$

The roadmap for the internship is then

- Build estimators of the parameters (μ, α) from the observations of a Hawkes process N with intensity $\lambda^{(\mu,\alpha)}$ using maximum likelihood estimators.
- Study the later estimators using the ergodic properties of N_t when H_0 is satisfied. Namely that
$$\frac{N_t}{t} \xrightarrow[t\to\infty]{\mathbb{P},H_0} \frac{\mu}{1-1/\alpha}$$
and the associated Central Limit Theorem $\sqrt{t}(\frac{N_t}{t} - \frac{\mu}{1-1/\alpha}) \xrightarrow[t\to\infty]{d,H_0} \mathcal{N}(0,\sigma^2)$. The aim being characterizing the limit distribution under H_0 of the quantity
$$\sqrt{t}\left(\frac{\hat{\mu}}{1-1/\hat{\alpha}} - \frac{\mu}{1-1/\alpha}\right)$$
- Propose a testing procedure for H_0 and study, for different alternative hypothesis H_1, its performances.

Extensions to more general functions h are then possible. The same roadmap can be investigated; the estimation of α is then replaced by the more difficult problem of estimating the L^1 norm of h.

References

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[2] Anna Bonnet, Charlotte Dion-Blanc and Maya Sadeler Perrin. Testing procedures based on maximum likelihood estimation for Marked Hawkes processes ArXiv preprint 2410.05008, 2024.

[3] Laub, Patrick J. and Lee, Young and Taimre, Thomas *The Elements of Hawkes Processes*, 2021. 10.1007/978-3-030-84639-8 Springer International Publishing.