Master's thesis proposal

Modelisation of new dispersion phenomena in photonic and phononic crystals.

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A photonic crystal is a nanostructure characterised by a periodic variation in its refractive index. In these media, the propagation of light is modeled by Maxwell's equations and the spectral theory of periodic operators applies. The spectrum of the underlying operator consists of bands (frequencies at which waves propagate) separated by gaps (where propagation is 'forbidden'). Thus, photonic crystals are used as optical filters in the bands and as mirrors in the gaps.

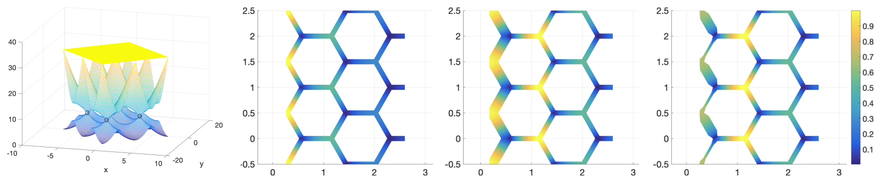

Figure 1: Conical point in the dispersion surfaces (gauche), localized modes (right)

A major advance in this field concerns topological insulators, in which the geometry (via symmetries) of the crystal plays a key role. Like graphene in solid-state physics, topological insulators in photonics are periodic structures that, for certain frequency ranges, behave as insulators in their volume but allow guided modes to propagate along a line defect in the crystal. Among these crystals, those with hexagonal symmetry have remarkable properties (see Fig. 1). On the one hand, some of their dispersion surfaces touch each other conically at points called Dirac points. On the other hand, by breaking the symmetry of the crystal, a gap can be opened at the Dirac points and, after a local perturbation, guided modes can be created: surface waves localised transversely to the defect. Unlike other structures, their existence is resistant to local variations in the defect. These are referred to as topologically protected modes. For certain models, an equality has been demonstrated between a topological index, the Chern number (associated with the crystal), and a spectral flux, which counts the number of guided modes in a gap [4, 5].

In this master's thesis, we propose to study the persistence or non-persistence of Dirac points under small perturbations of hexagonal periodic media. We will study structures that are perturbations of reference media, allowing for explicit calculations (see Fig. 2). Here, we propose to analyse, using asymptotic methods, fine structures [2] that can be approximated by graphs and media composed of two materials with high permittivity contrast [1].

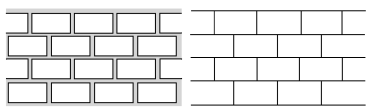

Figure 2: Perturbation d'un milieu hexagonal

The internship work focuses on theoretical approaches, but depending on the candidate's areas of interest, digital illustrations may also be produced. It is possible to pursue a PhD programme after completing the internship.

Desired experience

We are looking for a candidate with a strong background in partial differential equations. Good knowledge of spectral theory and asymptotic analysis, as well as programming skills, are desirable but not mandatory.

Lieu du stage:

Villetaneuse (Université Sorbonne Paris Nord) or Marseille (Aix-Marseille Université).

References

[1] M. Cassier and M. Weinstein, High contrast elliptic operators in honeycomb structures, SIAM Journal MMS 19 (4) (2021), pp. 1784-1856.

[2] B. Delourme and S. Fliss, Guided modes in a hexagonal periodic graph like domain, SIAM Journal MMS 22 (3) (2024), pp. 1196-1245

[3] S.-A. Nazarov, Opening of a gap in the continuous spectrum of a periodically perturbed waveguide, Mat. Zametki, 87(5):764–786, 2010.

[4] A. Drouot. The bulk-edge correspondence for continuous honeycomb lattices. Communications in Partial Differential Equations, 2019

[5] D. Gontier, Edge states in ordinary differential equations for dislocations. Journal of Mathematical Physics, 61(4):043507, 2020.