Simulation of displacements and gravity variations associated with sources in a volcanic context

This project is in the framework of an ANR aimed at developing methods to jointly analyze satellite deformation data (InSAR) and gravity variation data measured on active volcanoes, such as Piton de la Fournaise. Our goal is to better discriminate the nature, location, and geometry of volcanic activity sources, and to better anticipate magma recharge and eruptions. To account for heterogeneous 3D elastic domain, simulations will be conducted using fictitious domain methods, which are faster than traditional finite element methods. These methods are originally designed for approximating fluid-structure interaction problem.

From a geometric perspective, a volcano is represented by a domain $\Omega \subset \mathbb{R}^3$, whose boundary is decomposed into $\partial\Omega = \Gamma_N \cup \Gamma_D$. A fracture is considered within the domain and is represented by a surface $\Gamma_C \subset \Omega$ (see Figure 1). This fracture undergoes a magma intrusion, which will be modeled by a force term (i.e., a "Neumann" type condition).

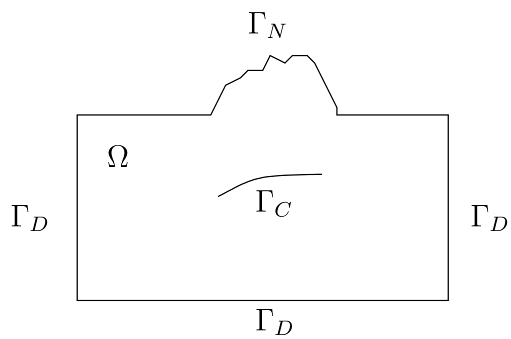

Figure 1: Domain Ω

In this internship, we propose to build on a fictitious domain method previously developed in [1, 2] to simulate surface displacements induced by fractures and to couple it with a method for calculating gravity variations induced by mass influx into the fracture. The gravity field is assumed to depend on a potential whose variation depends on the mass influx and the deformation of the volcanic structure (see [3, 4]).

More precisely, the objective will be to calculate the variation of the gravity field:

$$\delta g = -\partial_z \phi_g + \gamma_{FA} \boldsymbol{u} \cdot \boldsymbol{e}_z \text{ on } \Gamma_N, \tag{1}$$

where γ_{FA} is the gravity gradient on Γ_N, the surface of the ground, and u and ϕ_g satisfy the system of partial differential equations below (see Figure for the domain Ω). The deformation field

$\boldsymbol{u}$, with values in $\mathbb{R}^3$, satisfies the linear elasticity system:

$$\begin{cases} -\mathbf{div}\,(\sigma\,(\boldsymbol{u})) & = 0 \quad \text{in } \Omega, \\ \sigma\,(\boldsymbol{u}) \cdot \boldsymbol{n} & = \boldsymbol{t} \quad \text{on } \Gamma_C, \\ \sigma\,(\boldsymbol{u}) \cdot \boldsymbol{n} & = 0 \quad \text{on } \Gamma_N, \\ \boldsymbol{u} & = 0 \quad \text{on } \Gamma_D, \end{cases} \tag{2}$$

where $\sigma(\boldsymbol{u})$ denotes the stress tensor given by Hooke's law:

$$\sigma(\boldsymbol{u}) = \lambda\,\varepsilon(\boldsymbol{u}) + \mu\,\mathrm{Tr}(\varepsilon(\boldsymbol{u})), \quad \varepsilon(\boldsymbol{u}) = \frac{1}{2}\left(\nabla u + \nabla u^\top\right).$$

In this system, the vector field $\boldsymbol{t}$ is a traction force acting on the fracture Γ_C, which is a surface included in the domain Ω. The physical parameters λ and μ, depending on the elastic material considered, are given. The gravitational potential is the solution to the elliptic equation:

$$\begin{cases} -\Delta\phi_g & = 4\pi G\,(-\mathrm{div}\,(\rho\boldsymbol{u}) + \delta\rho_m)\,\mathbf{1}_\Omega(x) \quad \text{in } \widehat{\Omega}, \\ \phi_g & = 0 \quad \text{on } \partial\widehat{\Omega}, \end{cases} \tag{3}$$

where $\mathbf{1}_\Omega(x)$ is the indicator function of Ω and $\widehat{\Omega}$ is a box including Ω to approximate the condition at infinity $\lim_{\|x\|\to+\infty} \phi_g(x) = 0$. Here, $\delta\rho_m$ is a source term related to the material influx in the magma intrusion Γ_C, and G is the universal gravitational constant.

The tasks to be performed during this internship will be as follows:

- Study the theoretical aspects (existence, uniqueness, and continuity with respect to the data) of the systems (2) and (3).
- Program the solution of the Poisson equation and integrate it into the existing code aimed to determine the displacement field $\boldsymbol{u}$. The Python programming language will be used, interfaced with the finite element library GetFEM++.

This project is interdisciplinary between mathematics and geophysics, so the candidate must be curious about geophysical aspects and volcanism. The project will take place at the Camille Jordan Institute on the campus of Jean-Monnet University in Saint-Étienne.

Useful information:

- Contacts: Olivier Bodart (olivier.bodart@univ-st-etienne.fr) and Niami Nasr (niami.nasr@univ-st-etienne.fr).
- Duration: 6 months.
- Start date: February 2025.
- Compensation: At the current hourly rate (€4.35/hour)

References

[1] Oliver Bodart, Valérie Cayol, Farshid Dabaghi, and Jonas Koko. An inverse problem in an elastic domain with a crack: a fictitious domain approach. *Computational Geosciences*, pages 1–13, 2022.

[2] Olivier Bodart, Valérie Cayol, Sébastien Court, and Jonas Koko. Xfem-based fictitious domain method for linear elasticity model with crack. *SIAM Journal on Scientific Computing*, 38(2):B219–B246, 2016.

[3] Maurizio Battaglia and P Segall. The interpretation of gravity changes and crustal deformation in active volcanic areas. *Geodetic and geophysical effects associated with seismic and volcanic hazards*, pages 1453–1467, 2004.

[4] Gilda Currenti, Ciro Del Negro, and Gaetana Ganci. Modelling of ground deformation and gravity fields using finite element method: an application to etna volcano. *Geophysical Journal International*, 169(2):775–786, 2007.