Sujet de stage M2 et thèse 2025-2028

Towards more efficient kinetic and hybrid (kinetic-fluid) models and codes

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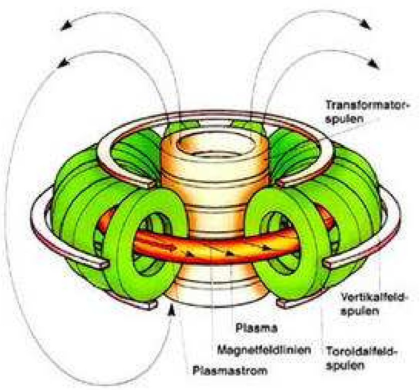

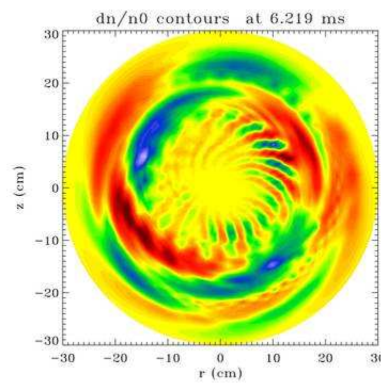

Figure 1: Left: Tokamak; Right: Turbulences in the density of the plasma

Summary

This PhD-thesis will focus on the **theoretical as well as numerical aspects of the turbulent plasma dynamics in a tokamak** (see Fig. 1). The subject is related to the magnetically confined, thermonuclear fusion processes (ITER project) with the aim to contribute to the research-field of the development of new sources of energy, able to satisfy the requirements of safety, low environmental impact and unlimited availability of resources. The controlled thermonuclear fusion, based on an efficient magnetic confinement of very hot plasmas, is at the moment one of the most promising concepts. Classical plasmas, characterized by low densities and high temperatures, are the core of this physical field. The **mathematical framework** of this PhD project will be essentially based on a **kinetic** description of the plasma dynamics. Discussions with the IRFM-team at the CEA-Cadarache will permit to remain close to reality.

Context

Plasmas are much more than a gas constituted of charged particles. Collective effects play an important role and the underlying physics is very different from that of neutral gases. The behaviour of a plasma is very complex, the main reasons being the nonlinear and self-consistent nature of the coupled system charged-particles $\leftrightarrow$ fields and the multiscale nature of the plasma dynamics.

Fusion plasmas are weakly collisional, due to the high temperatures and the low densities, such that the kinetic framework is the most appropriate approach for their detailed description. In particular this amounts to solve a coupled system of two Vlasov or Fokker-Planck equations for the ion/electron distribution functions $f_{i,e}$

$$\begin{cases} \partial_t f_i + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_i + \dfrac{q}{m_i} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{v}} f_i = Q_{ii}(f_i) + Q_{ie}(f_i, f_e) \\ \partial_t f_e + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_e - \dfrac{q}{m_e} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{v}} f_e = Q_{ee}(f_e) + Q_{ei}(f_e, f_i)\,, \end{cases}$$

coupled to Poisson's equation for the electrostatic potential (or Maxwell's equations in the electromagnetic case)

$$-\Delta\phi = \frac{q}{\varepsilon_0}(n_i - n_e)\,, \quad \mathbf{E} = -\nabla\phi\,.$$

The difficulty with a fully kinetic treatment is however the high-dimensionality of the phase-space ($6D$). Furthermore the presence of multiple spatio-temporal scales makes the problem even more complicated, rendering it inaccessible for numerical simulations. In our particular case, apart the strong magnetic field, it is the small mass ratio $m_e/m_i \approx 10^{-4}$ of the particles which induces disparate scales and hence difficulties; for a typical tokamak plasma with similar electron and ion temperatures the electron dynamics is faster than the ion dynamics, the ratio of the thermal velocities being given by $v_{th,e}/v_{th,i} = \sqrt{\frac{m_i}{m_e}} \approx 10^2$. This fact poses rather restrictive time-step constraints related to the fast electron motion, when a standard discretization of the bi-kinetic system is used, meaning that the numerical stability requires a CFL-condition of the type $v_{th,e}\Delta t \leq \Delta x$.

Another phenomena introducing multiple scales in the problem, is the presence of highly energetic particles in the plasma gas, such as α-particles (ions) or runaway electrons. The "core region" of thermonuclear tokamak is mainly constituted of thermal ions and electrons, characterized by Maxwellian distribution functions (and called in the following *bulk plasma*). Due to instabilities one observes the occurrence of highly energetic particles, which are characterized by the fact that their kinetic energy is much higher than the thermal energy of the bulk plasma and their density is much lower, namely one has

$$E_h = \frac{1}{2} m\,|v_h|^2 \gg k_B\,T_c\,, \qquad n_h \ll n_c\,,$$

where the index "h" stands for the energetic (hot) population and the index "c" for the thermal (cold) population. These energetic particles interact with the thermal bulk plasma and have various impacts on the overall behaviour of the fusion plasma. This PhD will focus on this specific energetic particle problematic.

Outline of the PhD

In this PhD thesis, we will investigate (from a theoretical as well as numerical point of view) new mathematical models as well as numerical algorithms for a more efficient plasma dynamics description, starting from a kinetic approach. As already mentioned, a fully kinetic description of the whole electron-ion plasma is very precise, however for the moment still out of reach in the full coupled $6D$ phase-space. Furthermore, such a fully kinetic system contains too many irrelevant spatio-temporal scales for the study of the plasma processes we intend to investigate. To redress this situation, a hybrid macroscopic-kinetic approach has to be adopted, eliminating the unnecessary fast dynamics and keeping the complete low-frequency physics. Such a reduced model is obtained via an asymptotic analysis, letting some specific parameters tend towards zero.

The aim of this PhD project is to introduce a mathematical multiscale description of the electron/ion dynamics of a magnetically confined tokamak plasma, separating the thermal and the energetic particles, which behave differently. The distribution function of the thermal particles is close to a Maxwellian distribution in velocity (fluid approach), whereas the energetic particles are far from being in a thermal equilibrium (kinetic approach). The first aim is to obtain via an adequate scaling procedure and a subsequent asymptotic limit, a hybrid kinetic-fluid model, the kinetic equation describing the fast, collisionless particles, and the fluid description describing the slow particles, which had the time to reach an equilibrium, through the collisions with the bulk. The second step is then to construct a performant numerical scheme, being able to follow at the discrete level this asymptotic limit. This should result in much cheaper simulations, as fluid models are far less expensive than using full kinetic models. During all this procedure, mathematical rigorous proofs shall sustain and permit to better understand the methodology.

Required knowledge

To cope with this thematic, the PhD student needs some basic knowledge of physics, functional analysis, PDEs, numerical methods, simulation languages.