

PÉRIODE D'ACCREDITATION : 2022 / 2026

UNIVERSITÉ PAUL SABATIER

SYLLABUS MASTER

Mention Mathématiques et applications

Master 1 MAT ESR parcours ESR - Enseignement
Agrégation

<http://www.fsi.univ-tlse3.fr/>
[http://departement-math.univ-tlse3.fr/
master-mention-mathematiques-et-applications-620690.kjsp](http://departement-math.univ-tlse3.fr/master-mention-mathematiques-et-applications-620690.kjsp)

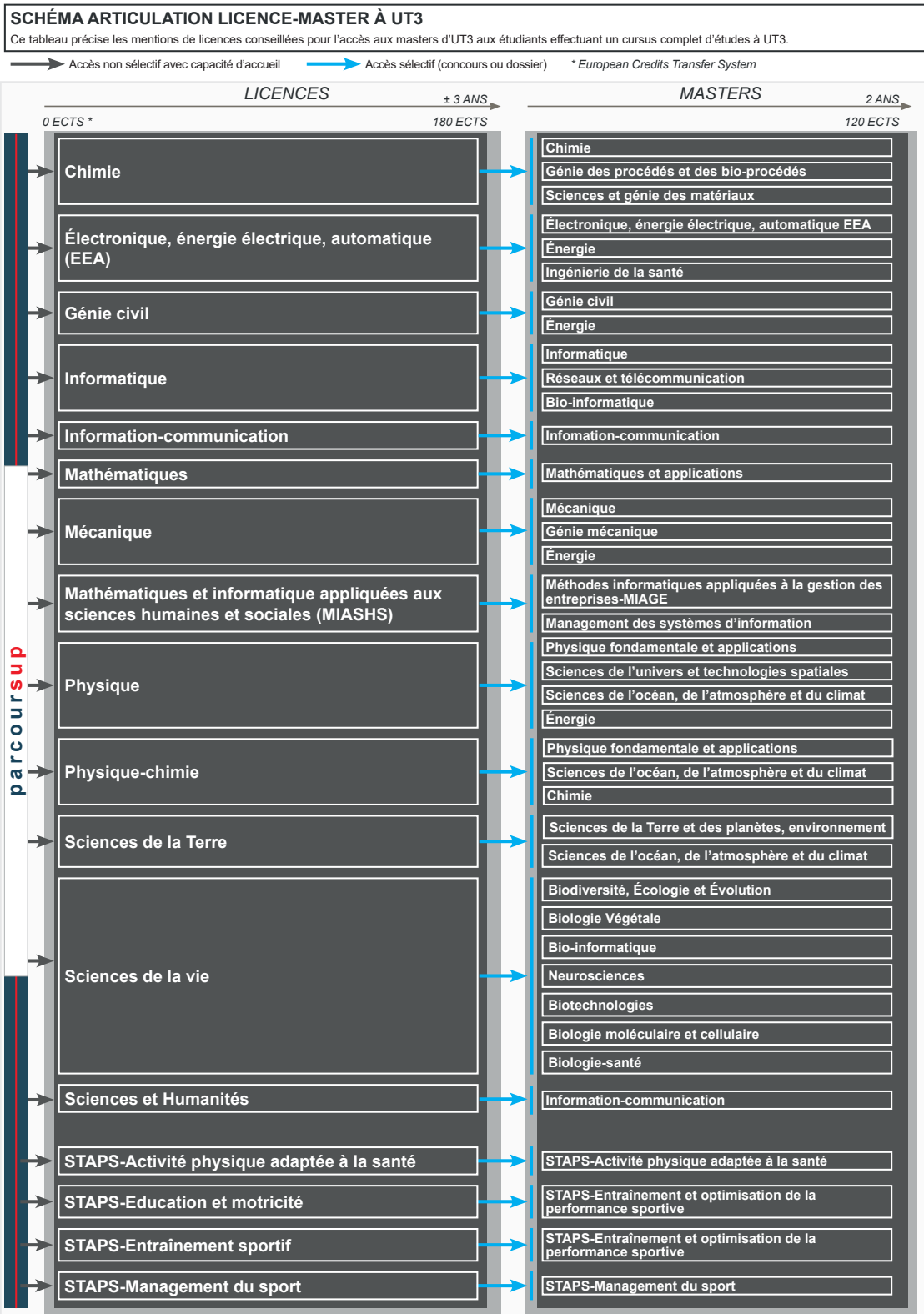
2022 / 2023

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DIAGRAM OF LINKS BETWEEN BACHELOR AND MASTER DEGREES



Toutes les mentions de licence permettent la poursuite vers des parcours du Master MEEF qui sont portés par l'Institut National Supérieur du Professorat et de l'Éducation (INSPE) de l'Université Toulouse II - Jean-Jaures.

Sources : Arrêté d'accréditation UT3 du 31 août 2021 et Arrêté du 31 mai 2021 modifiant l'arrêté du 6 juillet 2017 fixant la liste des compatibilités des mentions du diplôme national de licence avec les mentions du diplôme national de master. <https://www.legifrance.gouv.fr/jorft/td/JORFTEXT000043679251> et arrêté d'accréditation UT3

PRESENTATION

PRESENTATION OF DISCIPLINE AND SPECIALTY

DISCIPLINE MATHÉMATIQUES ET APPLICATIONS

The goal of the master "Mathematics and Applications" is to train mathematicians for careers in three possible directions :

mathematical engineering (tracks MApl3, SID, RO, SE, RI), research, innovation and development (tracks RI, RO, MApl3) and teaching (track ES).

The number of students trained in mathematics in France is much lower than the number of job offers, so that the career perspectives in mathematics are excellent.

SPECIALITY

The M1 Enseignement Supérieur et Recherche (ESR) is the first year of two courses of the Master in Mathematics and Applications : "Enseignement et Agrégation (EA)" and "Research and Innovation (RI)".

Courses are taught in English and French.

The M2 EA is taught entirely in French, while the M2 RI is taught entirely in English.

The EA program prepares students for the Agrégation external mathematics exams, with the main outlet being teaching in higher education (CPGE, universities) or in secondary schools.

The IR program prepares students for a doctoral thesis, possibly in connection with a company, with the main outlet being research in the academic world or in companies.

PRESENTATION OF THE YEAR OF MASTER 1 MAT ESR PARCOURS ESR - ENSEIGNEMENT AGRÉGATION

This first year is intended for students (male neutral) wishing to broaden and deepen their training in mathematics in order to acquire the knowledge necessary for the preparation of a Master 2 oriented towards research or innovation, for the preparation of the competitive examination for the agrégation, and more generally for any orientation requiring a solid background in mathematics.

The core of the program includes lectures with TD and/or TP. The evaluation includes a continuous assessment component, in the form of homework or assignments. The program for the first semester is fixed. In the second semester a choice of courses is offered, some jointly with the M1 MAPI3, with topics ranging from analysis to probability theory and statistics, from algebra to topology and geometry. The courses are taught by expert professors from the Department of Mathematics, which has very close ties with the renowned Toulouse Institute of Mathematics.

Two teaching units have a different format from the lectures. The reading seminar in the first semester operates on the principle of the flipped classroom, with student presentations and student-graded assignments. In the second semester, a research project provides an introduction to research in mathematics. Based on a book chapter or a research article, students work in relative autonomy with a supervisor and produce a thesis. This is defended and presented to the entire class.

At the end of the M1 program, students will have benefited from a very broad training in higher mathematics. They will have worked on their communication and knowledge transmission skills and will have been introduced to research in mathematics. They will then have all the keys in hand to specialize during the M2.

A selection is made at the entrance of the Master. To be admitted, it is necessary to have validated the equivalent of a third year of a Bachelor's degree in general mathematics.

LIST OF RECOMMENDED SPECIALITIES :

Licence Mathématiques parcours Enseignement Supérieur et Recherche (ESR)

CONTACTS SECTION

CONTACT INFORMATION CONCERNING THE SPECIALTY

PERSON IN CHARGE OF TEACHING AFFAIRS OF MASTER 1 MAT ESR PARCOURS ESR - ENSEIGNEMENT AGRÉGATION

BOISSY Corentin

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TABLE SUMMARIZING THE MODULES THAT MAKE UP THE TRAINING PROGRAM

page	Code	Title of the module	semestre*	ECTS	Mandatory Optional	Cours	e-Cours	e-TD	TD	TP	Mentoring EUR	Projet
First semester												
10	KMAR7AAU	ALGÈBRE (Algbr)	I	6	O	24			32			
11	KMAR7ABU	ANALYSE FONCTIONNELLE (FuncAna)	I	6	O	24			32			
12	KMAR7ACU	GÉOMÉTRIE DIFFÉRENTIELLE (DiffGeo)	I	6	O	24			32			
13	KMAR7ADU	PROBABILITÉS ET STATISTIQUES (ProbasStats)	I	6	O	24			32			
Choose 1 module among the following 2 modules :												
14	KMAR7AEU	SEMINAIRE DE LECTURE (ReadingSeminar)	I	6	O							100
15	KMAR7AFU	ANALYSE COMPLEXE - UE DE LICENCE	I	6	O							
	KMAXIN08	Analyse complexe 1 (An8-1)				14			14			
16	KMAXIN09	Analyse complexe 2 (An8-2)				14			14			
17	KMAR7FRU	MISE À NIVEAU (RefresherCourse)	I	0	F				24			
Second semester												
Choose 3 module among the following 7 modules :												
18	KMAR8AAU	GÉOMÉTRIE ET ALGÈBRE (GeoAlg)	II	6	O	24			32			
19	KMAR8ABU	STATISTIQUES	II	6	O	24			24	12		
20	KMAR8ACU	MÉTHODES NUMÉRIQUES POUR LES EDPS	II	6	O							
	KMAX8AC1	Méthodes numériques pour les EDPS				18			24	20		
	KMAX8AC2	Méthodes numériques pour les EDPS (projet)										12,5
21	KMAR8ADU	TOPOLOGIE ET ALGÈBRE (TopAlg)	II	6	O	24			32			
22	KMAR8AEU	TRANSFORMÉE DE FOURIER (FourierTransform)	II	6	O	24			32			
23	KMAR8AFU	PROCESSUS STOCHASTIQUES (StochProces)	II	6	O	24	0,01	0,01	32			
24	KMAR8AGU	EQUATIONS DIFFÉRENTIELLES ORDINAIRES ET EDP (ODE-PDE)	II	6	O	24			32			
25	KMAR8AHU	MODÉLISATION (Mod)	II	6	O	12			24	24		
26	KMAR8AKU	PROJET (Project)	II	3	O							50

* **AN** :year long teaching, **I** : first semester, **II** : second semester

page	Code	Title of the module	semestre*	ECTS	Mandatory Optional	Cours	e-Cours	e-TD	TD	TP	Mentoring EUR	Projet
Choose 1 module among the following 2 modules :												
27	KMAR8AVU	ANGLAIS	II	3	O				24			
28	KMAR8AZU	FRANCAIS LANGUE ETRANGERE	II	3	O				24			

* **AN** :year long teaching, **I** : first semester, **II** : second semester

LIST OF THE MODULES

UE	ALGÈBRE (Algbr)	6 ECTS	1st semester
KMAR7AAU	Cours : 24h , TD : 32h	Teaching in anglais	Personal work 94 h
Furrow(s) :	Sillon 3		
URL	https://moodle.univ-tlse3.fr/course/view.php?id=4187		

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TEACHER IN CHARGE OF THE MODULE

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LEARNING GOALS

The goal of this module is to study of the basic algebraic structure of fields and groups.

The field of complex numbers is obtained by incorporating a root of the equation $X^2+1=0$ to the field of reals numbers. This well known construction is in fact a very particular case of a much more general construction, and the relationships among the resulting fields turns out to be essential to understand many algebraic questions. An historical example is given by the Ruler and Compass constructions questions.

Groups are essentially designed to act on other objects. A lot of information about their structure can be learned from studying their linear actions on vector spaces : this observation gives rise to the theory of group representations and characters, which is studied in this module.

SUMMARY OF THE CONTENT

Field theory

- Reminders about fields
- Extensions
- Algebraic and transcendent elements
- Finite extensions, algebraic extensions. Application : Ruler and compass constructions, Wantzel theorem.
- Rupture field of an irreducible polynomial, algebraic closure
- Decomposition field, Normal extensions
- Finite Fields

Group representation theory

- Reminders on group actions
- Linear representations
- Irreducible representations
- Complex representations of finite groups
- Characters theory for finite groups, abelian and non-abelian

More detailed contents are available on the course Moodle page.

PREREQUISITES

Algebraic structures : groups, rings, fields. Polynomials : roots, Euclidean division, irreducible polynomial. Reduction of linear endomorphisms over \mathbb{C} .

REFERENCES

- « Algebra » by S. Lang.
- « Cours d'Algèbre » by R. Godement
- « Basic Algebra » by Nathan Jacobson

KEYWORDS

Fields, field extension, algebraic elements, algebraic closure.

Groups, group representation, characters of a group.

UE	ANALYSE FONCTIONNELLE (FuncAna)	6 ECTS	1st semester
KMAR7ABU	Cours : 24h , TD : 32h	Teaching in anglais	Personal work 94 h
Furrow(s) :	Sillon 7		
URL	https://moodle.univ-tlse3.fr/course/view.php?id=4339		

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LEARNING GOALS

The aim of this course is to acquire a mastery of tools that are fundamental for the understanding of results in the main fields of mathematics (geometry, probability, partial differential equations) as well as in applications of mathematics, in physics or mechanics. The basic notions of the topology of metric spaces with in particular the notions of complete spaces and compact spaces will be presented. We will focus on the study of normed vector spaces with in particular the fundamental example of function spaces. The rest of the course will be devoted to Banach spaces and to the study of Hilbert spaces which are an extension to infinite dimension of Euclidean spaces.

SUMMARY OF THE CONTENT

Chapter 1 Metric Spaces : Usual topologies of spaces. Notions and results of completeness, compactness. Banach and Hilbert spaces. Continuous linear applications. Some classical functional spaces

Chapter 2 Complete metric spaces : Baire's theorem. The Banach-Steinhaus theorem. Open application and closed graph theorems

Chapter 3 Compact metric spaces : Compact, sequentially compact spaces. General concepts and results. Arzelà-Ascoli theorem. Density theorems (Stone-Weierstrass). Compact linear operators.

Chapter 4 Duality. Hahn-Banach theorems : Analytic and geometric Hahn-Banach theorem. Duality and weak convergence in a Hilbert space (in TD).

Chapter 5 Hilbertian analysis : Projection on a closed convex, orthogonal, Hilbertian bases, Bessel and Parseval, Fourier series of locally integrable functions (Riemann-Lebesgue, Fejer, Dirichlet) in TD. Riesz representation theorem, weak convergence, weak sequential compactness of the unit ball of a Hilbert. Lax-Milgram and applications. Spectral theory of compact self-adjoint operators on a Hilbert space.

PREREQUISITES

Topology, Measure theory, Differential calculus

REFERENCES

H. Brézis, Analyse fonctionnelle - Théorie et applications

M. Eidsiedler, T. Ward, Functional Analysis, Spectral Theory, and Applications

P. D. Lax, Functional Analysis

KEYWORDS

espace métrique, complétude, compacité, théorèmes de Banach, analyse hilbertienne

UE	GÉOMÉTRIE DIFFÉRENTIELLE (DiffGeo)	6 ECTS	1st semester
KMAR7ACU	Cours : 24h , TD : 32h	Teaching in anglais	Personal work 94 h
Furrow(s) :	Sillon 3		
URL	https://moodle.univ-tlse3.fr/course/view.php?id=4338		

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LEARNING GOALS

The objective of the course is to explore the notion of differential subvariety and to generalize differential calculus in this framework, by introducing the notion of differential form and the associated integral calculus.

It allows a practical application of the differential calculus seen during the bachelor's degree through the use of

- the local inversion theorem to find several equivalent definitions of subvarieties,
- the quadratic forms to compare the relative position with the tangent space,
- the Stokes theorem and the construction of intrinsic invariants.

SUMMARY OF THE CONTENT

Part 1 : Plunging curves in \mathbb{R}^n (2.5 weeks)

Recall of differential calculus (implicit functions, local inversion) ; Euclidean isometries ; Parametric curves, metric study of curves : curvilinear abscissa, length, curvature ; Global rigidity of plane and left curves (curvature, torsion)

Part 2 : Surfaces of \mathbb{R}^3 (4.5 weeks)

Parametric surfaces, surfaces of revolution, ruled surfaces ; First fundamental form notion of area ; Application of Gauss ; orientable surfaces ; second fundamental form ; Principal curvatures ; Gaussian curvature, mean curvature ; Egregium theorem of Gauss ; Intrinsic distance, geodesics ; Euler characteristic, Gauss-Bonnet theorem

Part 3 : Differential subvarieties of \mathbb{R}^n (5 weeks)

Submanifolds of \mathbb{R}^n , tangent space, vector fields ; Differentiable applications, plunging, Sard's lemma ; Differential forms on \mathbb{R}^n (outer product, pull back and differential) ; Integration of differential forms, Stokes formula ; Abstract manifolds : atlas, examples, diffeomorphisms

PREREQUISITES

Multilinear algebra, differential calculus

TARGETED SKILLS

After validation, the student will be able to compare certain geometric objects on several scales :

- infinitesimal, via multilinear algebra
- local, via differential calculus
- global, via the interaction between geometry and topology

The student will have reinforced their knowledge of differential calculus and will be able to distinguish certain characteristic geometric properties.

REFERENCES

- M.DoCarmo, Differential Geometry of curves and surfaces. Prentice Hall (1976)
- M.Spivak, Differential Geometry Vol II, Publish or Perish Inc (1979)
- F.Warner, Foundations of differential manifolds and Lie groups. Springer (1983)

KEYWORDS

Curves, curvatures, diffeomorphisms, differential forms, isometries, submanifolds, surfaces

UE	PROBABILITÉS ET STATISTIQUES (ProbasS-tats)	6 ECTS	1st semester
KMAR7ADU	Cours : 24h , TD : 32h	Teaching in anglais	Personal work 94 h
Furrow(s) :	Sillon 8		
URL	https://moodle.univ-tlse3.fr/course/view.php?id=4337		

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LEARNING GOALS

This course is balanced between probability and statistics. It aims at presenting the theory of Markov chains with countable spaces and at presenting basic parametric and non-parametric statistics. The prerequisites are those of the L3 course in probability and measure theory. The notions taught in this module are intended to prepare students for the French "agrégation" and to give them a solid basis for further research in probability and statistics.

SUMMARY OF THE CONTENT

1. Reminders

Probability space. Convergence in law, almost sure, in probability, L^p . Borel Cantelli's lemma. LGN (strong), TCL. Optional : Kolmogorov's 0-1 law, complete proof of the strong law of large numbers

2. Markov chains with countable space

Definitions. Inhomogeneous Markov chain. Chapman Kolmogorov relation, transition matrix. Communication class. Classification of states. Stationary measure (definitions, existence and uniqueness, convergence, ergodic theorem). Reversibility. Applications.

3. Nonparametric statistics

Statistical model. Empirical measure. CLT-LLN applied to the empirical distribution function (Glivenko Cantelli, convergence to law, Kolmogorov test)

4. Parametric statistics

Gaussian vectors (recalled). Multivariate CLT. Delta method. Slutsky. Estimators. Confidence interval and confidence ellipsoid. Chi2 test. Parametric test (notion of level and power). Exponential families. Convergence of the maximum likelihood estimator (Fisher information matrix). Optional : Student. Method of moments. Exhaustive and complete statistics. Improvement of estimators.

PREREQUISITES

Measure theory, basic probability theory

REFERENCES

Barbe, Ledoux, Probabilité. De la Licence à l'Agrégation.

Foata, Fuchs, Processus de Poisson, chaînes de Markov et Martingales. Norris, Markov Chains.

Fourdrinier, Statistique Inférentielle 2ème Cycle. Van der Waart, Asymptotic Statistics.

KEYWORDS

Limit theorems, Markov chain, Statistical estimation

UE	SEMINAIRE DE LECTURE (ReadingSeminar)	6 ECTS	1st semester
KMAR7AEU	Projet : 100h	Teaching in anglais	Personal work 150 h
Furrow(s) :	Sillon 5		

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LEARNING GOALS

In this UE, students will be engaged in a "flipped classroom". Topics are identified around central themes already seen in the undergraduate program. Students will be required to study and report on the chapters of these books through weekly lectures.

SUMMARY OF THE CONTENT

The scientific content will depend on the texts chosen by the teachers. Examples could be : linear algebra, integration in several variables, probability.

SPECIFICITIES

Flipped classroom : no lectures, student presentations, tests graded by other students.

TARGETED SKILLS

- Presenting to an audience
- Build a presentation
- Questioning, giving constructive criticism
- Master the content of the bachelor's program

KEYWORDS

flipped classroom, seminar

UE	ANALYSE COMPLEXE - UE DE LICENCE	6 ECTS	1st semester
Sous UE	Analyse complexe 1 (An8-1)		
KMAXIN08	Cours : 14h , TD : 14h	Teaching in anglais	Personal work 94 h
Furrow(s) :	Sillon 5		

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TEACHER IN CHARGE OF THE MODULE

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UE	ANALYSE COMPLEXE - UE DE LICENCE	6 ECTS	1st semester
Sous UE	Analyse complexe 2 (An8-2)		
KMAXIN09	Cours : 14h , TD : 14h	Teaching in anglais	Personal work 94 h
Furrow(s) :	Sillon 5		

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TEACHER IN CHARGE OF THE MODULE

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UE	MISE À NIVEAU (RefresherCourse)	0 ECTS	1st semester
KMAR7FRU	TD : 24h	Teaching in anglais	Personal work 24 h

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LEARNING GOALS

Reminder of some of the notions from Bachelor's program, during the week before the start of the courses.

SPECIFICITIES

Optional and without evaluation.

UE	GÉOMÉTRIE ET ALGÈBRE (GeoAlg)	6 ECTS	2 nd semester
KMAR8AAU	Cours : 24h , TD : 32h	Teaching in anglais	Personal work 94 h

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LEARNING GOALS

- be comfortable with elementary calculations in homogeneous coordinates in a projective space, acquire an overview encompassing linear algebra and affine geometry
- discover examples of complex varieties with various geometries (rational, elliptic, and hyperbolic curves)
- deepen one's knowledge of holomorphic functions
- acquire a good understanding of tensor algebra, and know how to apply it (for example to the notion of differential form, linear representations of groups, extension of scalars, vector product, etc.)

SUMMARY OF THE CONTENT

- 1 Projective Geometry
 - 1.1 Projective spaces
 - 1.2 Link with Affine Geometry
 - 1.3 Examples
 - 2 Varieties of small degrees
 - 2.1 Study of conics and quadrics
 - 2.2 Planar cubics and complex tori
 - 2.3 And then (cultural part, without proof)
 - 3 Tensor algebra
 - 3.1 Tensor algebra
 - 3.2 Exterior algebra
 - 3.3 Symmetric product
- Detailed syllabus on the Moodle page of the course.

PREREQUISITES

linear algebra, affine geometry, holomorphic functions

REFERENCES

Excerpts from :

Perrin, La géométrie projective linéaire ; Audin, Géométrie ; Audin, Un cours sur les fonctions spéciales ; Harris, Algebraic Geometry - A first course ; Debarre, Introduction à la géométrie algébrique ; Debarre, Algèbre 1

KEYWORDS

projective geometry, conics and quadrics, elliptic curves, tensor algebra

UE	STATISTIQUES	6 ECTS	2 nd semester
KMAR8ABU	Cours : 24h , TD : 24h , TP : 12h	Teaching in anglais	Personal work 90 h

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TEACHER IN CHARGE OF THE MODULE

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LEARNING GOALS

The principal objective is to acquire notions and crucial properties of popular models parametric and non-parametric statistics :- The general linear model, the basic model for multivariate dependencies, which will be generalized in Master 2 by recent method about models penalized by a class of functions.

- non-parametric signed-rank tests, with minimal hypotheses on the distribution of the data.

- study of time series, the basic model for trend researchs, with a glimpse on models which are not stationnary.-

The various modelisations and methods presented will be illustrated during pratical sessions (Travaux Pratiques) on R, Scilab, Matlab ou Python.

SUMMARY OF THE CONTENT

- The general linear model, and generalisations - 5 weeks. Gaussians vectors, linear transforms, Cochran's theorem. General linear model, method of least squares, method of Maximum likelihood. regression coefficients in multiple regression. Significance testing (Student), testing nested models (Fisher), Prediction interval. Variance analysis with one or two factors.
- Non-parametric tests - 3 semaines. Recall of enumerative combinatorics and discrete probability, distribution of order statistics, Wilcoxon rank-sum test.
- Time series - 4 semaines. Trends and seasonal patterns of a time series. Stationnarity, estimation of the autocorrelation function, Portemanteau test. White noise, removal of an underlying trend, step detection. Stationnary series, partial autocorrelation (AR, ARMA).

PREREQUISITES

- Basic knowledge in statistics (estimation and test), integration and probability, linear algebra.

REFERENCES

- "Probabilités, analyse des données et statistique", Gilbert Saporta.
- "Le modèle linéaire par l'exemple", Jean-Marc Azais et Jean-Marc Bardet.
- <https://www.python.org/>

KEYWORDS

- Linear model, tests, time series

UE	MÉTHODES NUMÉRIQUES POUR LES EDPS	6 ECTS	2nd semester
Sous UE	Méthodes numériques pour les EDPs		
KMAX8AC1	Cours : 18h , TD : 24h , TP : 20h	Teaching in anglais	Personal work 88 h

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TEACHER IN CHARGE OF THE MODULE

NEGULESCU Claudia

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LEARNING GOALS

Non stationary linear PDEs

- Classification of the partial differential equations and study of their basic qualitative properties of each type of equations
- Classical finite volumes or finite differences schemes for linear scalar equations.
- Numerical analysis of these schemes : stability, consistency and convergence

At the end of this teaching, the student will know how to

- recognize the nature of a PDE,
- show the well-posedness of a problem
- design an algorithm to approach the solution
- implement this algorithm and will also have a critical eye on the results obtained.

Labs in Python

SUMMARY OF THE CONTENT

1. Heat equation, 5 weeks :

- 1.1 introduction with complex systems. Ex : Navier-Stokes
- 1.2 Resolution with Fourier in multi-D
- 1.3 Approximation by Finite difference or finite volumes

2. Transport equation, 5 weeks :

- 2.1 Introduction with complex systems (ex : Euler)
- 2.2 Resolution of the linear transport equation, with the characteristic curves, problem of boundary conditions (specific of hyperbolic problems)
- 2.3 Approximation by finite difference or finite volume methods, only in 1-D, explicit and implicit schemes, stability Linf and L2, consistency, convergence.

3. Wave equation. 2 weeks :

- 3.1 Introduction with complex systems (ex : Helmotz, Maxwell)
- 3.2 Approximation by finite difference or finite volume methods,

PREREQUISITES

M1 MAPI3 : being at ease with the contents of the PDE course of the 1st semester and ODE.

M1 EA/RI : course ODE-PDE strongly recommended to follow in parallel

REFERENCES

- L. C. Evans, Partial Differential Equations, American Math. Soc., 1999.
- P.A. Raviart and JM Thomas. Introduction à l'analyse numérique des équations aux dérivées partielles

KEYWORDS

Parabolic, hyperbolic, linear partial differential equations. Finite differences or finite volumes.

UE	TOPOLOGIE ET ALGÈBRE (TopAlg)	6 ECTS	2 nd semester
KMAR8ADU	Cours : 24h , TD : 32h	Teaching in anglais	Personal work 94 h

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LEARNING GOALS

- Knowing basic topology (topological spaces, continuous functions, compactness, connexity) and main examples (manifolds, simplicial complexes).
- Being able to study products and quotients of topological spaces (in particular in the case of group actions)
- Being able to tell two non homotopic functions
- Being able to compute the fundamental group and use its functorial behavior
- Knowing the notion of covering space and use it. Understanding the action of the fundamental group on the universal covering, with a clear understanding of the case of finite graphs.
- Knowing the definition of smooth manifold and the classification of surfaces.

SUMMARY OF THE CONTENT

- Topology : basics, induced topology on a subspace, continuous functions and homeomorphisms. Reminders on metric spaces. Axioms of separation. Examples : varieties, graphs, simplicial complexes, affine algebraic varieties and Zariski topology.
- CW-complexes and Euler characteristic
- Elements on groups : presentations, amalgam of groups.
- Seifert-van Kampen theorem (without proof).
- Notions of category theory : definition and examples (Ens, Gr, Vec, Top). Functors. The fundamental group is a functor.
- Coverings. Property of raising paths and homotopies.
- The fundamental group of S^1 is \mathbb{Z} .
- Criterion for raising applications (without proof).
- The universal covering (definition and construction), action of the fundamental group. Examples : covering of graphs.
- Differentiable variables and differentiable functions
- Tangent space and differential of a function
- Critical points and values, Sard's Lemma (without proof)
- Degree of an application and its homotopic invariance of degree (without proof) ; index of a singularity of a vector field
- Poincaré-Hopf and Brown's theorems
- Classification of surfaces (Euler characteristic and statement).

PREREQUISITES

Naive set theory, Notions of general topology, Metric spaces, Elementary analysis in \mathbb{R} and \mathbb{R}^n , Groups, group morphisms

REFERENCES

J. R. Munkres, Topology (second edition) Pearson New Int. Ed. 2014.

Alan Hatcher, "Algebraic topology" (<https://pi.math.cornell.edu/~hatcher/>)

John Milnor, \mathbb{O} Topology from the differentiable viewpoint \mathbb{O} , Princeton University Press 1965

KEYWORDS

Topological spaces, homotopy, fundamental group, covering maps, surfaces

UE	TRANSFORMÉE DE FOURIER (FourierTransform)	6 ECTS	2nd semester
KMAR8AEU	Cours : 24h , TD : 32h	Teaching in anglais	Personal work 94 h
URL	https://moodle.univ-tlse3.fr/course/view.php?id=4336		

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LEARNING GOALS

The aim of this course is twofold. On the one hand, it completes the integration course of the bachelor's degree by introducing two fundamental tools : the convolution product and the Fourier transform. On the other hand, we introduce the notion of distribution, in order to be able to derive a usual function in the sense of distributions. We will also extend the convolution product and the Fourier transform to distributions. All these tools will already allow us to discuss some simple examples of partial differential equations. Moreover, in order to be able to compute explicitly elementary solutions of PDEs, we will prove Stokes' and Green's formulas for regular functions, results which also have their own interest for other fields of mathematics.

SUMMARY OF THE CONTENT

Chapter 1 : Convolution product.

General properties ; Convolution product in Lebesgue spaces ; Applications to the regularization of functions

Chapter 2 : Fourier transform

Schwartz class ; Fourier transform in L1 ; Link with derivation, Fourier transform in Schwartz class ; Fourier transform in L2

Chapter 3 : Stokes formula, Green's formula

Lebesgue measure on a hypersurface ; Stokes formula on a regular open ; Green's formula

Chapter 4 : Distributions

Space of test functions ; Distributions, examples ; Derivation of distributions ; Distributions with compact supports ; Convolution of distributions ; Tempered distributions, Fourier transform of distributions

Chapter 5 : Introduction to Sobolev spaces (optional, if there is time).

PREREQUISITES

Lebesgue integration, Differential calculus

REFERENCES

Th. Gallouët, R. Herbin : Mesure, intégration, probabilités

J.M. Bony : Cours d'analyse, Théorie des distributions et analyse de Fourier

C. Zuily : Éléments de distributions et d'équations aux dérivées partielles

KEYWORDS

convolution product, Fourier transform, distributions

UE	PROCESSUS STOCHASTIQUES (StochProces)	6 ECTS	2nd semester
KMAR8AFU	Cours : 24h , TD : 32h , e-Cours : 0,01h , e-TD : 0,01h	Teaching in anglais	Personal work 94 h
URL	https://moodle.univ-tlse3.fr/course/view.php?id=3741		

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LEARNING GOALS

This course is an introduction to stochastic processes. It presents the tools of the theory of martingales as well as the Poisson process with applications. This course aims at preparing students for the "agrégation" exam (especially for option A of the exam) and for further studies in the field of probability and statistics.

SUMMARY OF THE CONTENT

Part 1 : Conditional expectation

- Conditional expectation and distribution, discrete variables, L2, positive, L1

Part 2 : Martingales

- Sub-/Supermartingale, Doob decomposition theorem
- Uniform integrability
- Optional stopping theorem (proof for bounded stopping times)
- Maximum inequality
- Convergence theorems for a.s., L2 and L1 martingales (L2 proof)
- Examples of non-uniformly integrable martingales,
- Application to the law of large numbers (exercice)

Part 3 : Poisson processes

- Definitions. Different characterizations. Application to queueing theory.

PREREQUISITES

Basic measure theory and probability theory. Course on Markov chains and statistics (first semester of M1)

REFERENCES

Philippe BARBE et Michel LEDOUX, Probabilité. De la Licence à l'Agrégation. Editions Espaces 34, EDP Sciences (2007).

Dominique FOATA, Aimé FUCHS. Processus de Poisson, chaînes de Markov et Martingales. Cours et exercices corrigés Dunod.

KEYWORDS

Conditional expectation, martingales, Poisson process

UE	EQUATIONS DIFFÉRENTIELLES ORDINAIRES ET EDP (ODE-PDE)	6 ECTS	2nd semester
KMAR8AGU	Cours : 24h , TD : 32h	Teaching in anglais	Personal work 94 h
URL	https://moodle.univ-tlse3.fr/course/view.php?id=3740		

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LEARNING GOALS

The aim of this course is first to make students familiar with the basic elements of the theory of ordinary differential equations (Cauchy theory, qualitative behavior, notions of stability) by illustrating them on examples from physics or biology for example. In a second step, we study the transport phenomena modeled by linear hyperbolic partial differential equations by means of the method of characteristics. We then introduce the notion of weak solutions for this type of problem. Finally, in a third step, we set up the functional analysis tools necessary to study elliptic boundary problems on a bounded interval.

SUMMARY OF THE CONTENT

Chapter 1 : Ordinary Differential Equations

- Global Cauchy-Lipschitz theorem
- Linear ODEs
- Flow of a vector field
- Local Cauchy-Lipschitz theorem
- Equilibria. Stability. Asymptotic stability
- Detailed study of examples. Phase portraits

Chapter 2 : Transport equations

- Transport models in 1D
- Classical solutions of transport equations. Characteristics.
- Optional : Weak solutions of transport equations.

Chapter 3 : Variational formulations of elliptic boundary problems

- The elastic string model
- Variational formulations. Lax-Milgram theorem
- Sobolev spaces

PREREQUISITES

Differential Calculus, Integration, Topology and Basic Functional Analysis

REFERENCES

Sylvie Benzoni-Gavage, Calcul différentiel et équations différentielles, 2010. Florent Berthelin, Equations différentielles, 2017. Haïm Brézis, Analyse fonctionnelle, 1999. Françoise Demengel et Gilbert Demengel, Espaces fonctionnels, 2007

KEYWORDS

Differential equations. Partial differential equations

UE	MODÉLISATION (Mod)	6 ECTS	2nd semester
KMAR8AHU	Cours : 12h , TD : 24h , TP : 24h	Teaching in anglais	Personal work 90 h
URL	https://moodle.univ-tlse3.fr/course/view.php?id=3723		

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LEARNING GOALS

This is an introduction to deterministic and random modeling, that is, the development of mathematical models that allow the study of real life problems, situations or objects. This module thus provides the bases for the preparation of the French « agrégation » (options A and B). The concepts are also fundamental for research in applied mathematics.

SUMMARY OF THE CONTENT

Part 1 (random modeling) :

- Reminder of classical laws for modeling, representation of laws of probabilities and samples
- Generation of random variables
- Illustration of almost sure convergence and convergence in law, application to a classical model
- Markov chains : use in modeling, simulation, invariant measures and applications
- Parametric statistics : parametric estimation, confidence intervals, tests. Asymptotic study and initiation to work with real data

Part 2 (deterministic modeling) :

- Reminders on ordinary differential equations
- Reminders on the discretization of ordinary differential equations and their numerical resolution
- Illustration using classical problems : the simple and damped pendulum, epidemiological model, prey-predator model, excitable system, neural networks, Lorenz model, etc.
- Around transport equations : modeling, some theoretical results, discretization using the finite difference method and numerical simulations.
- Around the heat equation : modeling, some theoretical results, discretization using the finite difference method and numerical simulations.

For detailed syllabus, see Moodle page.

PREREQUISITES

Analysis and probability/statistics at the level of L3 - early M1. Basic knowledge of Python with scientific packages.

SPECIFICITIES

The course is structured in 2 parts : random and deterministic modeling. Each part includes 6 sessions, a session consisting of 3 hours of lessons / TD as well as 2 hours of practical work (divided into groups according to the number of students).

TARGETED SKILLS

This module provides an introduction to the following skills :

- establish simple models using notions of L3-M1 level of analysis and probabilities
- study the properties of these models by applying classical theorems
- simulate these models on the computer using the Python language.
- interpret the results obtained by calculation or simulation with a view to the initial problem.

REFERENCES

Bercu, Chafaï, Modélisation stochastique et simulation. Rivoirard, Stoltz, Statistique en Action. Ross, Introduction to Probability Models

Filbet, Analyse numérique. Goudon, Mathématiques pour la modélisation et le calcul scientifique

KEYWORDS

Mathematical modeling, simulation

UE	PROJET (Project)	3 ECTS	2nd semester
KMAR8AKU	Projet : 50h	Teaching in anglais	Personal work 75 h
URL	https://moodle.univ-tlse3.fr/course/view.php?id=4355		

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LEARNING GOALS

This module is an introduction to research in mathematics. Students carry out a research project, generally in pairs, under the supervision of a member of the Mathematics Department. It consists of the study of a text (book chapter, easy-to-understand article, lessons or texts from the agrégations) with possibly an exploratory part. The students write a thesis and defend it in front of the whole class.

SUMMARY OF THE CONTENT

Depending on the subject proposed by the supervisor.

SPECIFICITIES

- Independent work, usually in pairs, under supervision
- writing of a thesis
- defense in front of the whole class

Evaluation taking into account these three elements.

TARGETED SKILLS

- Initiation to research in mathematics through the study of a subject in autonomy
- Know how to write complex mathematical documents
- Know how to present a complex mathematical subject to an audience

KEYWORDS

research, autonomy, thesis, presentation

UE	ANGLAIS	3 ECTS	2nd semester
KMAR8AVU	TD : 24h	Teaching in anglais	Personal work 51 h

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TEACHER IN CHARGE OF THE MODULE

CHAPLIER Claire

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UE	FRANCAIS LANGUE ETRANGERE	3 ECTS	2 nd semester
KMAR8AZU	TD : 24h	Teaching in anglais	Personal work 75 h

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TEACHER IN CHARGE OF THE MODULE

GOFFINET Akissi

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GLOSSARY

GENERAL TERMS

DEPARTMENT

The departments are teaching structures within components (or faculties). They group together teachers lecturing in one or more disciplines.

MODULE

A semester is structured into modules that may be mandatory, elective (when there is a choice) or optional (extra). A module corresponds to a coherent teaching unit whose successful completion leads to the award of ECTS credits.

ECTS: EUROPEAN CREDITS TRANSFER SYSTEM

The ECTS is a common unit of measure of undergraduate and postgraduate university courses within Europe, created in 1989. Each validated module is thus assigned a certain number of of ECTS (30 per teaching semester). The number of ECTS depends on the total workload (lectures, tutorials, practicals, etc.) including individual work. The ECTS system aims to facilitate student mobility as well as the recognition of degrees throughout Europe.

TERMS ASSOCIATED WITH DEGREES

Degrees have associated domains, disciplines and specialities.

DOMAIN

The domain corresponds to a set of degrees from the same scientific or professional field. Most of our degrees correspond to the domain Science, Technology and Health.

DISCIPLINE

The discipline corresponds to a branch of knowledge. Most of the time a discipline consists of several specialities.

SPECIALITY

The speciality constitutes a particular thematic orientation of a discipline chosen by a student and organised as a specific trajectory with specialised modules.

TERMS ASSOCIATED WITH TEACHING

LECTURES

Lectures given to a large group of students (for instance all students of the same year group) in lecture theatres. Apart from the presence of a large number of students, lectures are characterized by the fact they are given by a teacher who defines the structure and the teaching method. Although its content is the result of a collaboration between the teacher and the rest of the educational team, each lecture reflects the view of the teacher giving it.

TD : TUTORIALS

Tutorials are work sessions in smaller groups (from 25 to 40 students depending on the department) led by a teacher. They illustrate the lectures and allow students to explore the topics deeper.

TP : PRACTICALS

Teaching methods allowing the students to acquire hands-on experience concerning the knowledge learned during lectures and tutorials, achieved through experiments. Practical classes are composed of 16 to 20 students. Some practicals may be partially supervised or unsupervised. On the other hand, certain practicals, for safety reasons, need to be closely supervised (up to one teacher for four students).

PROJECT

A project involves putting into practice in an autonomous or semi-autonomous way knowledge acquired by the student at the university. It allows the verification of the acquisition of competences.

FIELD CLASS

Field classes are a supervised teaching method consisting of putting into practice knowledge acquired outside of the university.

INTERNSHIPS

Internships are opportunities enabling students to enrich their education with hands-on experience and to apply lessons learned in the classroom to professional settings, either in industry or in research laboratories. Internships are strongly regulated and the law requires, in particular, a formal internship convention established between the student, the hosting structure and the university.

