Introduction to Spectral Theory

M2RI – 2022/23

Julien Royer

julien.royer@math.univ-toulouse.fr

In this course we will discuss basic tools of spectral theory. This generalizes in infinite dimension the analysis of the spectrum and the diagonalization of matrices, and the applications to linear ODEs. Spectral theory is used in all the areas of mathematics where linear opearators arise (PDEs, probability, numerical analysis, geometry, etc.).

1. Spectrum of some bounded operators
- Spectrum of bounded operators
- Bounded self-adjoint operators
- Compact operators, Fredholm operators
- Spectrum of compact operators

2. Unbounded operators
- Unbounded operators, closed operators
- Spectrum of unbounded operators, resolvent
- Discrete spectrum, essential spectrum
- Operators with compact resolvents

3. Operators in Hilbert spaces
- Adjoint
- Symmetric operators, selfadjoint operators
- Min-max principle
- Functional calculus

4. Semigroups and evolution equations
- Dissipative operators
- Continuous semigroups, generators
- Hille-Yosida Theorem
- Application to non-linear problems

5. Introduction to perturbation theory