

# An Introduction to K-theory for $C^*$ -algebras

proposed by Paulo CARRILLO ROUSE

The main goal of these lectures is to give a general introduction to the theory of  $C^*$ -algebras and to the K-theory functors for these algebras. This subject is at the intersection of some domains in analysis, algebra, geometry and topology. In these lectures the chosen approach is towards some applications in topology and geometrical analysis.

Topics covered:

- (1) Banach algebras and basics on spectral theory.
- (2) A brief introduction to  $C^*$ -algebras.
- (3) The Gelfand-Naimark theorem, that gives an explicit equivalence between the category of commutative  $C^*$ -algebras and the category of locally compact Hausdorff topological spaces.
- (4) Examples of  $C^*$ -algebras associated to discrete and (compact) Lie groups. Crossed products and other noncommutative spaces.
- (5) Topological K-theory of  $C^*$ -algebras. First properties and examples.
- (6) AF-algebras (Approximately finite algebras) and their classification via topological K-theory and traces.
- (7) Cohomological properties of the K-theory. Long exact sequences, Thom isomorphism, Bott periodicity, excision and Mayer-Vietoris for the topological spaces case.
- (8) Towards some applications in geometric analysis and topology. In particular, Fredholm index morphism for compact manifolds and statement of Atiyah-Singer index theorem that states a computation of the Fredholm index morphism that uses classic algebraic topology tools.

Prerequisites: Good background in topology and basic functional analysis. Non of the first semester lectures of the program is required to follow this course. It can be however interesting to link this lecture to the one on Spectral Theory and the one on differential/algebraic topology, depending on the audience we could use some the material seen in these two lectures.

Bibliography:

- Dixmier, Les  $C^*$ -algèbres et leurs représentations. (French) Deuxième édition Cahiers Scientifiques, Fasc. XXIX Gauthier-Villars Éditeur, Paris 1969.
- Murphy,  $C^*$  algebras and operator theory. Academic Press, Inc., Boston, MA, 1990.
- Rordam, An introduction to K-theory for  $C^*$ -algebras. (English summary) London Mathematical Society Student Texts, 49. Cambridge University Press, Cambridge, 2000.
- Skandalis, Lectures notes available at <https://webusers.imj-prg.fr/georges.skandalis/>
- Wegge-Olsen, K-theory and  $C^*$ -algebras. A friendly approach. Oxford Science Publications. The Clarendon Press, Oxford University Press, New York, 1993.