

# An introduction to sensitivity analysis

Master M2RI

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The context of the course is the study of complex phenomena represented by multivariate “black-box” functions. Such functions are typically simulators in industry, or cost functions of machine learning algorithms such as neural network, gradient boosting, etc. Sensitivity analysis aims at answering two main questions:

- Which input variables are the most influential on the output? Which ones are not influential?
- How can we quantify their influence on the output, alone or in interaction with other input variables?

The course will present some powerful mathematical tools of global sensitivity analysis:

## 1. Variance-based sensitivity analysis

- Sobol-Hoeffding (ANOVA) decomposition and variance-based (Sobol) indices,
- Estimation of sensitivity indices: Pick-Freeze technique, rank-based estimators,
- Extensions to multivariate and functional outputs,
- Beyond ANOVA: Cramér-von Mises indices, goal-oriented sensitivity indices, HSIC...

## 2. Gradient-based sensitivity analysis

- Gradient-based sensitivity indices (DGSM),
- Connexion between variance- and gradient-based indices with Poincaré inequalities,
- Active subspaces.

## 3. Complements

- ANOVA revisited with operators; Application to Möbius decomposition of copulas,
- Correlated inputs and Shapley indices,
- Metamodeling tools for costly functions: (polynomial) chaos.

## References

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- [3] Janon A., Klein, T., Lagnoux, A., Nodet, M., Prieur, C. (2014). Asymptotic normality and efficiency of two Sobol index estimators. *ESAIM: Probability and Statistics*, 18, 342-364.
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