AN INTRODUCTION TO HYPERBOLIC AND TRANSLATION SURFACES

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In this course, we will study Riemann surfaces via two geometric structures, hyperbolic surfaces and translation surfaces.

- In the first part of the course, we will review the basic notions of differential manifolds, metrics, coverings and Riemann surfaces.
- In the second part we will focus on the case of Riemann surfaces with negative Euler characteristic. Using the uniformization theorem (admitted), we will see that they inherit a hyperbolic metric. For compact surfaces, we will establish the pants decompositions and give a heuristic approach to Teichmüller spaces and Fenchel–Nielsen coordinates.
- The last part is an introduction to the study of translation surfaces. These are pairs (X, ω), where X is a compact Riemann surface and ω a holomorphic 1-form. They carry a flat metric with conical singularities. Such structures also naturally appear when studying billards in rational polygons. We will investigate geometry and dynamics in these structures, in relation to geometry and dynamics in their moduli space.

Prerequisites

Basic knowledge on holomorphic functions. Attendance of the M1 course Topologie et Algèbre is strongly required, although we will review essential materials as needed.

References

- J. Hubbard, *Teichmüller theory and applications to geometry, topology, and dynamics. Volume 1. Teichmüller theory*, Matrix Editions , 2006.
- A. Zorich, *Flat surfaces*, https://arxiv.org/abs/math/0609392
- J. Athreya and H. Masur, *Translation surfaces*, Graduate Studies in Mathematics 242 (AMS), 2024.